Radio Resource Management for Green Wireless Networks

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Abstract

In this thesis, we investigate two techniques used for enhancing the energy or spectral efficiency of the network. In the first part of the thesis, we propose to combine the network future context prediction capabilities with the well-known latency vs. energy efficiency tradeoff. In that sense, we consider a proactive delay-tolerant scheduling problem. In this problem, the objective consists of defining the optimal power strategies of a set of competing users, which minimizes the individual power consumption, while ensuring a complete requested transmission before a given deadline. We first investigate the single user version of the problem, which serves as a preliminary to the concepts of delay tolerance, proactive scheduling, power control and optimization, used through the first half of this thesis. We then investigate the extension of the problem to a multiuser context. The conducted analysis of the multiuser optimization problem leads to a non-cooperative dynamic game, which has an inherent mathematical complexity. In order to address this complexity issue, we propose to exploit the recent theoretical results from the Mean Field Game theory, in order to transition to a more tractable game with lower complexity. The numerical simulations provided demonstrate that the power strategies returned by the Mean Field Game closely approach the optimal power strategies when it can be computed (e.g. in constant channels scenarios), and outperform the reference heuristics in more complex scenarios where the optimal power strategies can not be easily computed.

In the second half of the thesis, we investigate a dual problem to the previous optimization problem, namely, we seek to optimize the total spectral efficiency of the system, in a constant short-term power configuration. To do so, we propose to exploit the recent advances in interference classification. the conducted analysis reveals that the system benefits from adapting the interference processing techniques and spectral efficiencies used by each pair of Access Point

(AP) and User Equipment (UE). The performance gains offered by interference classification can also be enhanced by considering two improvements. First, we propose to define the optimal groups of interferers: the interferers in a same group transmit over the same spectral resources and thus interfere, but can process interference according to interference classification. Second, we define the concept of 'Virtual Handover': when interference classification is considered, the optimal Access Point for a user is not necessarily the one providing the maximal SNR. For this reason, defining the AP-UE assignments makes sense when interference classification is considered. The optimization process is then threefold: we must define the optimal i) interference processing technique and spectral efficiencies used by each AP-UE pair in the system; ii) the matching of interferers transmitting over the same spectral resources; and iii) define the optimal AP-UE assignments. Matching and interference classification algorithms are extensively detailed in this thesis and numerical simulations are also provided, demonstrating the performance gain offered by the threefold optimization procedure compared to reference scenarios where interference is either avoided with orthogonalization or treated as noise exclusively.

Résumé

Dans le cadre de cette thèse, nous nous intéressons plus particulièrement à deux techniques permettant d'améliorer l'efficacité énergétique ou spectrale des réseaux sans fil. Dans la première partie de cette thèse, nous proposons de combiner les capacités de prédictions du contexte futur de transmission au classique et connu tradeoff latence - efficacité énergétique, amenant à ce que l'on nommera un réseau proactif tolérant à la latence. L'objectif dans ce genre de problèmes consiste à définir des politiques de transmissions optimales pour un ensemble d'utilisateur, qui garantissent à chacun de pouvoir accomplir une transmission avant un certain délai, tout en minimisant la puissance totale consommée au niveau de chaque utilisateur. Nous considérons dans un premier temps le problème mono-utilisateur, qui permet alors d'introduire les concepts de tolérance à la latence, d'optimisation et de contrÃ´le de puissance qui sont utilisés dans la première partie de cette thèse. L'extension à un système multi-utilisateurs est ensuite considérée. L'analyse révèle alors que l'optimisation multi-utilisateur pose problème du fait de sa complexité mathématique. Mais cette complexité peut néanmoins être contournée grâce aux récentes avancées dans le domaine de la théorie des jeux à champs moyens, théorie qui permet de transiter d'un jeu multi-utilisateur, vers un jeu à champ moyen, à plus faible complexité. Les simulations numériques démontrent que les stratégies de puissance retournées par l'approche jeu à champ moyen approchent notablement les stratégies optimales lorsqu'elles peuvent être calculées, et dépassent les performances des heuristiques communes, lorsque l'optimum n'est plus calculable, comme c'est le cas lorsque le canal varie au cours du temps. Dans la seconde partie de cette thèse, nous investiguons un possible problème dual au problème précédent. Plus spécifiquement, nous considérons une approche d'optimisation d'efficacité spectrale, à configuration de puissance constante. Pour ce faire, nous proposons alors d'étudier l'impact sur le réseau des récentes avancées en classification

d'interférence. L'analyse conduite révèle que le système peut bénéficier d'une adaptation des traitements d'interférence faits à chaque récepteur. Ces gains observés peuvent également être améliorés par deux altérations de la démarche d'optimisation. La première propose de redéfinir les groupes d'interféreurs de cellules concurrentes, supposés transmettre sur les mêmes ressources spectrales. L'objectif étant alors de former des paires d'interféreurs "amis", capables de traiter efficacement leurs interférences réciproques. La seconde altération porte le nom de "Virtual Handover" : lorsque la classification d'interférence est considérée, l'access point offrant le meilleur SNR n'est plus nécessairement le meilleur access point auquel assigner un utilisateur. Pour cette raison, il est donc nécessaire de laisser la possibilité au système de pouvoir choisir par lui-même la façon dont il procède aux assignations des utilisateurs. Le processus d'optimisation se décompose donc en trois parties : i) Définir les coalitions d'utilisateurs assignés à chaque access point ; ii) Définir les groupes d'interféreurs transmettant sur chaque ressource spectrale ; et iii) Définir les stratégies de transmission et les traitements d'interférences optimaux. L'objectif de l'optimisation est alors de maximiser l'efficacité spectrale totale du système après traitement de l'interférence. Les différents algorithmes utilisés pour résoudre, étape par étape, l'optimisation globale du système sont détaillés. Enfin, des simulations numériques permettent de mettre en évidence les gains de performance potentiels offerts par notre démarche d'optimisation.

Contents

Acronyms

List of acronyms

AP	Access Point
BPC	Best Performance Configuration
BS	Base Station
CDMA	Code-Division Multiple Access
CoMP	Coordinated Multi-Point
CSI	Channel State Information
FPK	Fokker-Planck-Kolmogorov equation
GA	Genetic Algorithm
HJB	Hamilton-Jacobi-Bellman equation
iid	independent and identically distributed
IC	Interference Classification
ICT	Information and Communications Technology
ILP	Integer Linear Programming
IMS	Interference Management Strategy
JD	Joint Decoding
MC	Monte-Carlo
MCS	Modulation and Coding Schemes
MFE	Mean Field Equilibrium
MFG	Mean Field Game
NE	Nash Equilibrium
NLP	Non-Linear Programming
OPEX	Operational Expenditures
PDE	Partial Differential Equation
PDF	Probability Density Function
PPAD	Polynomial Parity Arguments on Directed graphs
QoS	Quality of Service

RRM	Radio Resource Management
SIC	Successive Interference Cancellation
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SRE	Spectral Resource Element
TS	Time Slot
UE	User Equipment
VH	Virtual Handover
wrt	with respect to

Remark. *All abbreviations are also re-defined at their first use in each chapter to facilitate partial reading of the manuscript.*

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Chapter 1

Introduction

1.1 Background and Motivations for Green Wireless Networks

1.1.1 Network Trends

With exponential increases in communication traffic, the Information and Communication Technology (ICT) industry currently accounts for 2% of worldwide carbon emissions, and that figure is expected to at least double over the next decade as more people seek to connect with each other and with more content in new, richer ways [1]. Even a 2% contribution to global emissions and energy consumption is significant: the network component of this represents some 250-300 million tons of carbon emissions, according to GreenTouch Green Measure [2]. Apart from the environmental concern, there is also an economical motivation behind reducing the network power consumption: For instance, it appeared that Vodafone's global energy consumption for 2007-2008 was about 3000 GWh, which corresponds to emitting 1.45 million tons of CO2 and represents a monetary cost of several hundred million Euros [3]. More specifically, it represents a significant part of the operating expenses of a network operator: for example, in a mature European market, it even reaches 18% [3]. But while network traffic is growing exponentially and is doubling every two years, the revenues of the network operators are only growing annually at less than 10% [1]. For this reason, a critical challenge that is facing the ICT industry is how to ensure that the operating cost, that are expected to follow the exponential communication traffic trend, can be reduced in order to keep pace with the slowly increasing revenues. Based on this context, over the past two decades, concepts like 'green communications' have emerged and designing energy-efficient communication networks has become an important issue, in particular, to manage operating costs [4]. The 'green communication' concept sets the aspiration of achieving a thousandfold improvement from 2010 levels in the future energy efficiency, over current designs for wireless communication networks [4, 5]. This challenge is also rendered nontrivial by the requirement to achieve this reduction without significantly compromising the quality of service (QoS) experienced by the network users. In order to enhance the global energy efficiency of the network, several research directions for green wireless networks have been identified. Since most of the energy consumption of a mobile network comes from the wireless access, i.e. comes from the power cost at each Base Stations (BS) side in the network, it appears immediately that the greatest opportunity to reduce energy consumption is to improve the base stations deployment and energy efficiency [6]. Several research directions for Green Wireless Networks have been proposed in literature, among which:

- Radio Resource Management (RRM) energy-efficient techniques, such as power control, sleep modes, etc.
- Improvement of the network deployment efficiency
- Multi-antenna techniques, such as using large multiple antennas system, virtual multiple input multiple output (MIMO), beamforming or spatial multiplexing.

In this thesis, we focus mainly on the first two concepts, that we detail more extensively hereafter.

1.1.2 Research Directions for Green Wireless Networks - Radio Resource Management, Cognitive Radios and Power Control

1.1.2.1 Radio Resource Management Techniques: Power Control and Sleep Mode

A first set of solutions suggests to enhance the energy efficiency of the network via Radio Resource Management (RRM) and power control techniques. In such techniques, made possible by the emergence of concepts like Cognitive Radios [7], the objective consists of adapting the network transmissions settings, to the network context, in order to maximize a given utility function. In green wireless networks, the utility function to be considered is often the energy efficiency of the network or the users. Among the possible settings to be adapted, the transmission powers used at each base station often appears as the best candidate for two reasons. First, the operating power cost of the BS is in fact directly related to the transmission power [8], which is in turn related directly to the power consumption and the energy efficiency of the network. Second, the transmission powers are directly related to the interference patterns in the network: adapting the powers allows to regulate the interference in the network. This leads to power control problems, whose objective is to regulate the power, in order to provide each user an acceptable connection, while at the same time reducing the total power consumption of the network and limiting the interference [9, 10]. It is well known, in literature, that minimizing interference using power control increases capacity, while reducing the power consumption at the same time [11, 9, 10]. Kandukuri and Boyd [12] also address both the minimization of transmitter power subject to constraints on outage probability and the minimization of outage probability subject to power constraints.

In practice, the power control approach can also aim at minimizing the power consumption of a base station, which can also be modeled with two parts. The first part describes the static power consumption, due to hardware cooling, the A/D conversion, the signal processing, etc. Depending on the load situation and the power consumption, a dynamic part adds to the static power [8]. It is observed that the static part is the main contribution to the base station power cost. As a consequence, turning the base station off, commonly referred to as sleep mode, when it is unused, might allow to save even more energy, as it almost completely negate the power cost of the base station [13, 14, 15]. In [16] for example, the authors then suggest to exploit this technique with high potential: they schedule transmissions in order to maximize the sleep time of the base station, thus greatly enhancing the energy savings.

1.1.2.2 Delay Tolerance and Future Knowledge: Proactive Delay-Tolerant Networks

At the same time, recent studies have also revealed that most of the network transmissions could be labeled as non-urgent: many mobile applications are then 'delay-tolerant', scheduling future transmissions can save power and thus money (often referred to as OPEX for Operational Expenditures): for example, a cell phone user might be willing to delay sending a non-urgent email message or download application updates for up to several hours if this allows to transmit over a low-cost interface [17, 18]. The system can then freely schedule its required transmission over the offered latency, in order to reduce the global power consumption required to complete this transmission. This is commonly referred to as the latency vs. energy-efficiency trade-off in literature [19] and it has led to the so-called concept of 'delay-tolerant networks' [20, 21, 22].

In such delay-tolerant networks, the system is allowed to schedule its transmission and adapt its transmission settings, often the transmission powers at each BS, in order to optimize a given utility function, under a set of given constraints. Both the utility function and the constraints are usually used to model the satisfaction of the users in the network, and/or the satisfaction of the network operator. In [23, 24, 25], the author study the activation problem which determines when a mobile will turn on in order to receive packets, and also address the transmission control problem, whose objective is to control the total power cost. The conducted optimization allows to maximize the throughput of the system, while constraining the energy to be used. Moreover, the delay tolerance can also be used to allow the system to better handle the network congestion, as suggested in [20, 26, 27]: the network can then freely decide to transmit when the network resources are underused, thus limiting the congestion.

In addition to the delay tolerance, several recent works have also revealed that human behavior was highly and accurately predictable [28, 29]. The mobility of a user in the network is often constrained by roads or streets, thus allowing for easily accurate short-term predictions on the user mobility [30, 31]. Coupling the prediction of the user mobility with radio maps, providing the expectation of the path loss perceived by a user at any geographical position, can lead to accurate predictions on the expected future link quality [32, 33, 34]. As a consequence, recent works have then looked forward to coupling scheduling techniques with future context predictions, in order to enhance the network performance leading to so-called proactive networks, as introduced in [35, 36, 37]. Significant diversity gains were analytically demonstrated, thus illustrating the significant potential benefit of proactive networks. In these papers [38, 39, 40] for example, the system is able to formulate predictions on the upcoming requests and user mobility: by coupling it with a radio map giving measured reception quality at different locations, the system can then formulate predictions on the expected future transmission contexts. And based on these predictions, it then adapts its present transmission settings, in order to limit its own outage probability.

1.1.3 Single User Proactive Delay-Tolerant Transmissions: a Toy Example for Convex Optimization

In the first half of this thesis, we investigate how the system might exploit an offered latency, when coupled with information about the upcoming transmission context, that we refer to as 'future knowledge'. To do so, we first consider an illustrative toy example of a proactive delay-tolerant system. In Chapter 3 and as in [41, 42], we investigate a single user system, where one base station is enforced to complete a given data transmission before a given deadline, but is able to adapt its transmission power settings and aims at minimizing the total power consumption required to complete the transmission before the deadline. The system can freely adapt the power level to be used at the beginning of each time slot. The remaining packet size decreases according to an instantaneous rate which is a function of the SINR, i.e. a function of the transmission power used, the current channel realization during this time slot. We assume that the system has perfect knowledge of the channel realization that will occur during the whole duration of the present time slot, and can then adapt the transmission setting to be used to the current channel, as well as the remaining packet size and the number of remaining time slots until the deadline. We also consider that the system has a certain knowledge about the future transmission context, more specifically it has a certain knowledge about the future channel realizations. The considered predictor is modeled as a Probability Density Function (PDF) that represents the prediction of the future channel realizations on each remaining time slot. The system can then adapt its current transmission power and instantaneous rate, based on both the present context and the expectation of the future transmission context.

In practice, defining the optimal power level to be used at the beginning of each time slot is equivalent to solving a power control problem, which consists of a mathematical optimization: the objective is to maximize or minimize a utility function (e.g. the energy cost, the energy efficiency,etc.) by systematically choosing the 'best' set of parameters for this function (e.g. the transmission power, as the utility function directly relies on the transmission powers at the base station). Most of the time, when facing power control and optimization problems, the problem turns out to be convex. We then may refer to the theory of convex optimization, for which the book of Boyd and Vandenberghe [43] is certainly one of the most cited and complete reference. In case of non-convexity, some papers either investigate how the problem can be assumed convex, or propose specific algorithms to deal with these non-convex scenarios. In the general case, the optimal solution is attained by computing the Lagrangian associated with the optimization problem. The Lagrangian links the objective function to equality and inequality constraints functions by using Lagrange multipliers. Karush-Kuhn-Tucker(KKT) conditions [44] can then be used to derive the optimal solution to the problem. When not possible, the alternative solution consists of an iterative backward dynamic programming algorithm, that we also detail in this chapter [45].

When the system has perfect a priori knowledge of the future yet to come, i.e. knows a long time in advance the exact future channel realizations, the optimal strategy can be simply computed using a time water-filling algorithm, which is derived from the Karesh-Kuhn-Tucker conditions [44]. Water-filling based power allocation techniques have been widely presented [43, 46, 47] and investigated in the literature [48, 49, 50]. However, accessing a perfect knowledge about the future is an ideal scenario.

In this chapter, we propose to investigate several scenarios of future knowledge, ranging from a complete lack of knowledge to a perfect knowledge scenario and observe how the system may benefit from each scenario of future knowledge. The scenarios investigated in this chapter can be either:

- **perfect knowledge:** the system has perfect a priori knowledge about the exact future channel realizations. This is the best future knowledge scenario that the system can be given and leads to the optimal performance bound.

- **zero knowledge:** the system has no information about the future channel realizations. In this scenario, the system either transmits at a constant rate on each time slot (equal-bit scheduler), or it transmits assuming the worst possible channel realizations for each remaining time slot (which in the end leads to a min-max problem, that can be solved using the time water-filling algorithm as well).

- **statistical:** the system is given the channels statistics. It can then compute the optimal power strategy to be used using the iterative backward dynamic programming algorithm.

- **short-term perfect knowledge, with zero information about the remaining time slots:** in this scenario, we assume that the system can perfectly predict the future channel realizations on a few upcoming time slots, but does not have any information about the remaining time slots.

- **short-term perfect knowledge, with statistical information about the remaining time slots:** in this scenario, we assume that the system can perfectly predict the future channel realizations on a few upcoming time slots. The system is also given the channel statistics as future knowledge about the channel realizations on the remaining time slots.

The complete list of future knowledge scenarios is extensively detailed in Section 3.4. In each scenario, numerical simulations provide good insights on how the system benefits from proactive resource allocation and each kind of future knowledge.

Through this simple illustrative example, we provide answers to the following three fundamental questions related to delay-tolerant networks and future knowledge:

- **How can the system exploit some future knowledge?** A possible way for the system to exploit this future knowledge relies on exploiting the power-efficiency latency trade-off. We model a delay-tolerant transmitter, and consider a power control optimization problem, where the objective is to minimize the global power consumption required for completing a fixed transmission before a given deadline. The transmitter is cognitive and can adapt its transmission power to the present transmission context, in real time. The decision process for the optimal power strategy is then affected by the present state (time remaining before deadline, packet size remaining,etc.) but is also able to take into account some piece of future knowledge about the future transmission context.

- **Does future knowledge offer significant performance gains?** The numerical simulations show that there is a significant gain between i) the zero knowledge scenario, which is the worst scenario of future knowledge, since the system does not know anything about the future transmission context, and thus is lower performance bound; and ii) the perfect knowledge scenario, which is the best scenario of future knowledge, since the system has perfect knowledge of the future at any time, and thus is the higher performance bound. Demonstrating that the gain was significant really mattered: if the performance gap had not been significant enough, then looking for future knowledge, and providing it to the system, so that it can exploit it via scheduling and proactive resource allocation would not have made sense. The performance gain would have been limited, and there would have been really little chance that this performance gain would have surpassed the cost of accessing and exploiting this future knowledge (commonly referred to as the 'cost of learning'). This chapter does not include details about how future knowledge might be acquired, nor does it define the cost of learning for every single future knowledge scenario. Nevertheless, a few details on this topic are discussed in Section 3.6.

- **What kind of future knowledge is really useful to the system?** The conducted analysis shows that the system may greatly benefit from partial future knowledge, and may almost reach the performance of the perfect knowledge scenario. More specifically, it turns out that a good statistical knowledge of the future context can offer significant performance gains. Also, it appears that a short-term knowledge (i.e. precise knowledge about the close future exclusively) can also provide significant performance gains.

1.1.4 Multi-user Proactive Delay-Tolerant Transmissions: Multi-user Non-Cooperative Stochastic Games

In the second chapter (Chapter 4) of the first half of this thesis, we investigate the extension of the previous toy example to a multiuser scenario. We consider $N \geq 2$ pairs of Base Stations and users, each BS is given the objective of transmitting a given packet (whose initial size may vary from one pair to another) to its assigned user before a common deadline. Each BS can again adapt the power level to be used at the beginning of each time slot, as in the previous chapter. The problem complexifies, as we must now consider interference, which models the competition between users. At each user receiver side, the SINR term now includes an interference term that sums up how the instantaneous rate of one pair is affected by the other pairs power strategies. We have then N competitive transmissions occurring, at the same time, and the BS have the same objective: completing a required transmission before a given deadline, at a minimal cumulated power cost. And each user decides at the beginning of each time slot, the optimal power strategy to be used for transmission, based on the current context (remaining packet size, remaining time slots, present channels realizations between all BS and users) and the expectation of the future channel realizations, which are modeled according to an Itô process [51].

In the considered multiuser competitive scenario, computing the optimal power strategy to be used by each user, at the beginning of each time slot relies on game theory, more specifically non-cooperative stochastic game theory, because of the ItÃ´ process. Game theory is a mathematical framework, born in the field of economics [52, 53], that investigates the strategical interactions between competing, rational decision takers known as players. Broadly speaking, game theory can be divided into cooperative game theory, in which players are free to form coalitions to achieve a common goal, and non-cooperative game theory, in which each player competes with each other to achieve a selfish goal [53]. In non-cooperative game theory, the most widely used solution concept is the famous notion of Nash Equilibrium (NE) [54, 53] and its refinements. A NE is an equilibrium state of the game in which no player can improve its utility by a unilateral deviation: a NE configuration satisfies all the players, as they do not feel like they could improve their situation by changing independently their current strategy, thus leading to a stable configuration.

When analyzing the NE configuration of the previously mentioned game, our conducted analysis reveals that two approaches can be considered, in the general case:

- The Nash Equilibrium configuration can be accessed by solving a set of N couple Partial Differential Equations (PDE), namely N coupled Hamilton-Jacobi-Bellman equations, as suggested in [53]. Solving a set of N coupled PDEs can rapidly become complicated, especially when the number of partial derivatives corresponds to all the possible transmission settings (all the cross channels between all BS and users, and the remaining packet sizes for each pair).

- When no stochasticity is considered, the Nash Equilibrium configuration can be approached by an iterative time water-filling algorithm [55, 56, 57, 58, 59], where each BS can adapt, its individual power strategy to the transmission context and the other BS current power strategies. However, when a BS adapts it power strategy, the interference pattern perceived by the other users in the system is reset and the other BS might no longer be satisfied with their current power strategy, and the readjustments will again reset the interference pattern of the system. Because of this 'ping-pong effect' between players, the iterative process is demonstrated to converge to a fixed point, which corresponds to the NE configuration [55]. However, the computation time required to observe such a convergence tends to explode when the number of users in the system N increases, rendering large problems untractable [60].

Both approaches appear complex because each player action has an immediate impact on the other players perceived performance. The decisions made by a player must then take into account the anticipation of the other users actions. This phenomenon dramatically increases the inherent mathematical complexity of the problem, especially when the number of users N grows large, but several solutions allow to bypass the complexity of the problem:

- Focus on scenarios where the number of users N remains small enough, so that we can solve the set of N coupled PDEs. This solution is however extremely limited in our scenario, as the set of equations becomes already extremely complex to solve, even for $N = 3$.

- Focus on scenarios where the evolution of the channels is simpler than a stochastic model. If the channel does not have a stochastic part (i.e. the channel evolution is perfectly estimated), the iterative time water-filling algorithm can be used to approach the NE configuration. However, we must keep in mind that the number of players in the system N must remain relatively small, so that the computation time necessary to observe the converge of the iterative algorithm to a fixed point, remains acceptable. In scenarios where the channels are constant wrt to time, as in [61] (i.e. both the deterministic and stochastic parts are equal to zero), the optimal power strategies can be simply computed by solving a set of N linear equations.

- A heuristic suboptimal power strategy can also be considered. Such a heuristic strategy is simple to compute, but is by definition suboptimal. In this chapter, we investigate two heuristics: a constant power heuristic (the power strategies are necessarily constant wrt time), and a full-power heuristic (which transmits at maximal power and stops when the transmission is completed).

Several solutions exist, but none of them allows to solve the problem in the stochastic configuration, with a large number of players N. This is quite problematic, especially since today's trend is to dense large heterogeneous networks, as detailed in Section 1.1.6.1.

1.1.5 Mean Field Games

We have demonstrated in the previous section, that the inherent mathematical complexity related to multi-user non-cooperative stochastic games could render the problem untractable, especially when the number of users grows large. The cause for this complexity is the high number of interactions between the N users in the system, N being supposed large. However, when the number of users grows large, the impact of a single player action on its neighbors might becomes negligible at the large scale of the system. Also, we might observe symmetries between users: same objective functions, same sets of actions, same evolution models, etc. It is then possible to simplify the problem, by exploiting those symmetries when the number of users N grows large [62]. The Mean Field Theory, relies on these ideas and allows to approximate a multi-user stochastic game and turn a N users game into a more tractable equivalent game, called a Mean Field Game (MFG), as it was introduced by Lasry and Lions [63, 64, 65]. This equivalent Mean Field Game presents the advantage of having a 2-body complexity only, compared to the N-body complexity of the initial multi-user non-cooperative stochastic game.

Several recent papers have implemented such a Mean Field framework, in order to simplify the resolution of multi-user stochastic games. For example, in [66, 67], every user has to adapt their strategies to the quality of their environment(link quality, channel, etc.), while ensuring a minimal SINR constraint. In [68], a similar and interesting analysis is provided, with an application of the MFG tools, into the topic of electrical vehicles in the smart grids. In [69, 61], the players are transmitters, who adapt their transmission powers to the quality of their link with the receiver, the strategies of the other users, and their battery level, while ensuring a SINR constraint. In a similar way, we turn an untractable N users stochastic game into a MFG and study the Mean Field Equilibrium of the new-built game. The Mean Field Equilibrium leads to the mean field optimal set of power strategies, that will be used for approximate the optimal strategies of the original N users stochastic game.

In Chapter 4, we propose to exploit the recent Mean Field Games advances, in order to transition our initial multi-user non-cooperative stochastic game into an equivalent Mean Field Game. The conducted analysis reveals how the Mean Field Equilibrium can be computed, in order to define a common power strategy to be used by any user in the initial multi-user non-cooperative stochastic game, in any configuration (time, remaining packet size, channels). The returned mean field power strategy approaches the optimal power strategy when it can be computed, for example in scenarios where there are no variations on the channels wrt time. We study the performance of the mean field power strategy, for several channel models (from the constant channel case to the complete stochastic problem) and we provide numerical simulations assessing the performance of the investigated optimal MFG, compared to a set of reference strategies (iterative time water-filling when possible, full-power heuristic strategy, constant power heuristic strategy).

Numerical results reveal that the Mean Field power strategy closely approaches the optimal power strategy, when it can be explicitly computed (i.e. constant channel scenarios). In scenarios where the channel become time-varying with no stochasticity, the optimal power strategy can not be simply computed for large dimension scenarios. Only constant power heuristic and full-power heuristic strategies are considered, and it is observed that the Mean Field power strategy outperforms both heuristics, revealing a twofold gain, that can be decomposed with one performance gain due to the latency and one performance gain due to the future knowledge. For this reason, proactive delay-tolerant frameworks appear to offer a significant energy gain, confirming the importance of the concept in Green Wireless Networks. It must noted that the future knowledge does not provide any gain, when the channels are not time-varying, as the equal-bit scheduler could have been used to compute the optimal power strategy, without any future knowledge, which confirms what has been observed in the previous chapter. Finally, we study the impact of the stochastic part on the Mean Field power strategy, and reveal that the uncertainty of the future strongly affects the optimal Mean Field power strategies: the system becomes more cautious about the future and might instead prefer to transmit notably earlier in advance, thus becoming less able to exploit the offered latency, as it did with the zero knowledge scenario, in Chapter 3. be decomposed with one performance gain due to the latency and one performance gain due to the future knowledge. For this reason, proactive delay-tolerant frameworks appear to offer a significant energy gain, confirming the importance of the concept in Green Wireless Networks. It must noted that the future knowledge does not provide any gain, when the channels are not time-varying, as the equal-bit scheduler could have been used to compute the optimal power strategy, without any future knowledge, which confirms what has been observed in the previous chapter. Finally, we study the impact of the stochastic part on the Mean Field power strategy, and reveal that the uncertainty of the future strongly affects the optimal Mean Field power strategies: the system becomes more cautious about the future and might instead prefer to transmit notably earlier in advance, thus becoming less able to exploit the offered latency, as it did with the zero knowledge scenario, in Chapter 3.

1.1.6 Research Directions for Green Wireless Networks - Enhancing the Deployment Efficiency and Bottlenecks

1.1.6.1 Enhancing deployment efficiency: towards large heterogeneous networks

A second set of solutions for green networking relies on improving the network deployment, by densifying it with relays, small cells, femto/pico-cells, leading to the so called dense heterogeneous networks [70, 71, 72]. This first set of solutions relies on the well-known fact that cell-size reduction is the simplest and most effective way to increase the global network capacity, by enhancing the spatial reuse [73]. Also, due to their short transmit-receive distance, small cells can greatly lower transmit powers, prolong handset battery life, and achieve a higher signal-to-interference-plus-noise ratio (SINR), thus resulting in a better spectral efficiency [72]. Improving the network deployment efficiency then leads to win-win solutions for network operators, as they enhance the global network capacity, while reducing at the same time the base station power costs.

However, it must be noted that such heterogeneous networks are more complex to handle for the operators, due to their large number of elements and their multi-tier topology structure, consisting of a first tier with high power large coverage macrocells and a second tier with low power small coverage pico/femto/small cells. The multiplicity of wireless communication systems increases the spectrum pollution due to interference. As a consequence of the large number of Base Stations transmitting in a same geographical area, the in-band interference, which models the interactions between the different elements of the networks sharing the same spectral resources, has become an important issue to be addressed in the heterogeneous networks.

1.1.6.2 Interference, The Universal Enemy

In the heterogeneous networks, the interference is classically perceived at each receiver side as the enemy. The classical approach treats interference, due to concurrent transmissions from other elements of the network using the same spectral resources, as an additional source of noise. For reliable transmissions to occur, the interference, which is classically treated as an additive source of noise, is perceived as an enemy. For this reason, it must be ideally avoided or at least strongly limited, in order to guarantee a good SINR, and a good spectral efficiency after interference processing. In order to cope with interference, two sets of techniques can be considered: techniques that aim at avoiding interference and techniques that aim at keeping the interference limited.

When attempting to avoid interference, the most common and simple approach consists of orthogonalizing transmissions, by enforcing the different elements of the network to transmit using different spectral resources. Several well-known resource allocation techniques have been proposed to achieve such orthogonalization, by simply separating signals in time, frequency, space or code: Time/Frequency Division Duplex (TDD/FDD), Time/Frequency Division Multiple Access techniques (TDMA/FDMA), Orthogonal Frequency Division Multiple Access (OFDMA) Code/Space Division Multiple Access (CDMA/SDMA) [74, 75]. However, such orthogonal resource allocation can not be met in dense networks, since the set of available resources might not be large enough to allocate exclusive resource blocks to each element in the network. Moreover, such orthogonal allocation techniques lead to poor spectral efficiencies and drives the system to a drastic suboptimal spectral efficiency operating point.

Due to the large number of elements in the network, and the limitation of spectral resources, the orthogonalization, even though simple, might not always be the best option. When the elements in the network share the same spectral resources, their transmission is limited by in-band interference. Nevertheless, resources can be shared and reused in a smart fashion, as long as we manage to mitigate the in-band interference. Such interference management is enabled by partial or full orthogonalization between competing interferers, as proposed by frequency reuse or graph coloring [76, 77, 78].

A second set of techniques, which allows interference to remain limited, so that reliable and efficient transmissions can occur, relies on the previously mentioned power control approaches. The approach consists of carefully adapting the transmission powers, so that interference remains under a target limit: the system carefully balances power budgets allocation among interfering sources, as it will be detailed in the first half of the thesis, or in multiple papers in literature [79, 10, 12, 11]. However, in order to solve such optimization problems and find optimal transmission strategies for every user in the system, we have to find an equilibrium configuration. When facing this problem, we demonstrated that this may involve a high mathematical complexity, especially when the system dimensions become large [80, 81, 60], as we must take into account all the one user to one user interactions, which is modeled by the interference perceived at each receiver side.

1.1.6.3 Is Interference Friend or Foe ? Interference Classification

Previously mentioned methods always assume that interference will be processed as an additive source of noise. In that sense, the mentioned methods do not profit of the recent advances in the domain of information theory, showing that interference might not necessarily be an opponent, but may become, in fact, an ally, especially in cases where the interference becomes strong, which is commonly identified as an interference badly compromising the transmission. In practice, Carleial [82] and, later on, Han & Kobayashi [83] have demonstrated that it was possible to exploit intrinsic properties of the interference, in order to process interference differently and obtain notably higher rates after interference processing. The observation, which allowed the trick to happen, consisted of observing that interference was not just an additive source of noise. In fact, Carleial suggested that, in scenarios of strong interference, the strong limitation of the rate was not due to theoretical limitations, but was instead due to the communications techniques employed for processing interference. He proposed an interference processing technique, which first aims at decoding the strong interference in presence of the primary signal, and then subtracts the decoded interference from the received signal, leaving it with no trace of interference. The main concept behind this idea is commonly referred to as Successive Interference Cancellation (SIC) [82] and is considered as the optimal interference processing technique for strong interference scenarios, as it allows to remove completely the strong interference. Based on this observation, it immediately appeared that a single interference mitigation technique, namely the noisy processing, could not perform well for all the possible scenarios of interference, ranging from weak to strong interference.

As a matter of fact, exploiting additional interference mitigation techniques, such as SIC, has been recently perceived as a promising feature for 5G networks [84]. It also inspired the works of Etkin & Tse [85], who investigated the different interference mitigation techniques from a single user point of view and their spectral efficiencies after interference processing. They defined the SNR/INR configurations for which each interference mitigation technique was the most-suited technique. In a two-user Gaussian interference channel, they proved that for any pair (R_1, R_2) in the interference capacity region, the considered schemes were able to achieve the spectral efficiencies pair (R_1-1, R_2-1) for any values of the channel parameters (i.e. SNR and INR). Basically, this means that the presented schemes in this paper were able to achieve spectral efficiencies within 1 bit/s/Hz of the capacity of the interference channel. Five interference regimes, i.e. interference mitigation techniques were identified, each of them being the most-suited technique in a given region of α, which was defined as $\alpha = \frac{log(INR)}{log(SNR)}$. In the following we recall the presented classification of the interference mitigation techniques proposed by Etkin and Tse, as the '5-Regimes Interference Classification'. We represented the 5 Regimes and their performance, illustrated by the well-known 'W-shaped' Figure 2.1.

This '5-Regimes interference classification' was later simplified by Abgrall [86, 87], who proposed a simplified version of this classification, reducing the classification to only 3 regimes. The interference may either be treated as an additive source of noise if it is perceived as weak, exploited and canceled via SIC if it is perceived as strong, or simply avoided via orthogonalization, if it is neither perceived as strong or weak. To justify the simplification, Abgrall suggested that even if some of them perform very well theoretically, they may suffer from infeasibility in practice because of excessive computational complexity or strict operating assumptions. For example, simultaneous superposition coding is up to now too complex to be used in practice. For this reason, Abgrall favored the use of techniques which can be implemented in practical systems without stringent limitations. More details about each classification and the considered classification regimes will be detailed in a short tutorial, in Section 5.2.

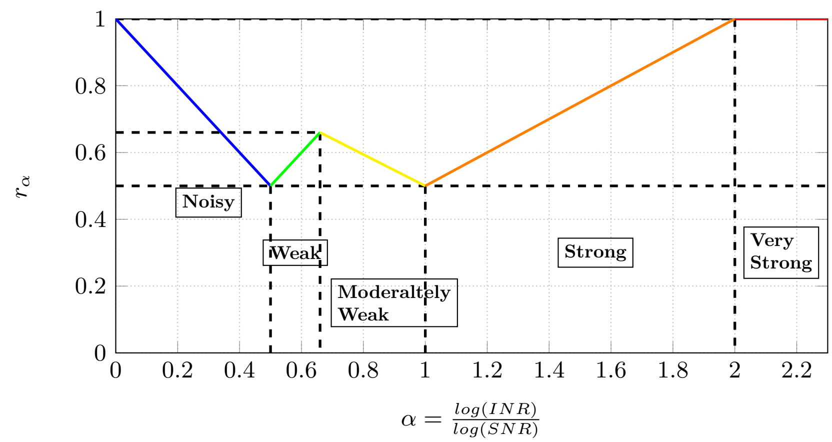

Figure 1.1: Generalized degrees of freedom, according to the α value. This 'W-shaped' curve exhibits an interference classification into 5 interference regimes.

In the second half of the chapter, we propose to address the dual problem to the previous power control approach, namely, we look forward to optimizing the spectral efficiency of the system, under a fixed power configuration. In this part, we propose to focus on the interference, which degrades the quality of the transmissions in the network. Similarly to recent works that have proposed to exploit interference classification (e.g. [88, 89]), we propose an interference classification based approach in Chapter 5, that enhances the system network performance, by finding the optimal way to process interference, thus exploiting the inherent properties of interference. To do so, we first consider a RRM problem, in a 2-users Gaussian Interference Channel. The system deals with the perception of the interference at each receiver side and aims at maximizing the total spectral efficiency, assuming the interference is treated according to the 3-regimes classifier defined by Abgrall. More specifically, we propose to adapt the perceived robustness of transmission at each receiver side, by adapting the spectral efficiencies and the reliability of the transmissions, to the channel context. This way, we reduce the complexity of the optimization problem, by only allowing changes on the interference perception of each user. We leave unchanged the short-term power configuration and interference patterns, since it causes an avalanche of changes in the network [60, 86]: such an approach directly tackles the 'ping pong effect' we described earlier and the associated computational complexity observed in the iterative processes and instead allows for low-complexity optimization. The analysis of the optimization problem reveals that, when maximizing the total spectral efficiency, interference does not have to be necessarily avoided or strongly limited: in fact, in this specific scenario, our study leads to a reduced interference classification, with 2 regimes for each user, that can be exploited in more sophisticated multi-user optimization problems. The two regimes are the noisy regime, used for weak interference scenarios and the SIC-based regime used in strong interference scenarios.

We also detail, in Section 6.4.2, the extension of the interference classification problem to a M-users Gaussian Interference Channel. Several papers, as [88], have tried to tackle the problem of both defining the best way to process interference in a multiple interferers scenario and estimating the system inherent spectral efficiency. Group SIC, iterative k-SIC or k-Joint Decoding approaches might also be considered as in [90], leading to multiple new regimes. Investigating these regimes requires to take into account the fact that decoding interference signals at each receiver is affected by the joint effect of interference, rather than each interfering signal. Therefore, it appears that it is better to consider directly the effect of the combined interference signal. Recently, interference alignment techniques have been proposed and are based on this principle. The use of these techniques leads to achievable spectral efficiencies that, in some cases, can be as good as those over the 2-user Gaussian interference channel [91, 92, 93]. However, these techniques remain theoretical and does not suit well practical implementation, which is the reason why we have not considered them in the conducted optimization. Instead, we propose a game-theoretic model for interference classification in M-users Gaussian Interference Channels. Even though suboptimal, the simple interference classification we propose can lead to notable results. It is discussed in Section 6.4.2.

1.1.7 Matching 'Friendly Interferers' Together, with Matching Theory

In Sections 5.4 and 5.5, we extend the initial problem of a M-users Gaussian Interference Channel, to a system with coalitions of multiple users per Base Station. The objective in this optimization consists of forming groups of interferers, with one interferer from each coalition. The interferers in a group of interferer transmit over the same spectral resources, and thus interfere, but can process interference according to our previous proposed interference classification for 2-users or M-users Interference Classification. The optimization problem can then be modeled as a one-to-one matching problem, with the objective of maximizing the total spectral efficiency after interference processing.

Matching theory [94, 95] has been especially influential in labor economics, where it has been used to describe the formation of new jobs, as well as to describe other human relationships like marriage: matching theory studies the resulting outcome when one or more types of searchers interact and aims at finding the best matching configuration that optimizes the outcome. In our scenario, the matching problem belongs to a special class of matching problems, namely M-Dimensions Multidimensional Assignment Problems (MAP), that can be solved optimally with the Kuhn-Munkres algorithm, when $M = 2$ [96, 97, 98]. The problem however becomes NP-Hard when $M > 2$ and a few suboptimal heuristic algorithms, mostly genetic algorithms have been proposed to cope with the mathematical inherent complexity of the problem [99].

1.1.8 Interference Classification and BS assignments: Graph Theory, Integer Linear Programming and Genetic Algorithms

In Chapter 6, we investigate the possibility of redefining the AP-UE assignments. When no interference classification was considered and interference was only processed as an additive source of noise, we could easily observe that the best AP to which a UE must be assigned to, is the one that provides the best SNR, as it will lead to the best spectral efficiency after interference being processed as noise. However, it is well-known in literature that when interference classification is considered, the best AP is not necessarily the AP providing the best SNR [100]. In Chapter 6, we then investigate the extension of the previous optimization detailed in Chapter 5, by assuming that the system may now re-define the AP-UE assignments at will.

The interference classification in the 2-users Gaussian Interference Channel is first updated, in order to take into account the possibility of defining the AP-UE assignments, along with the interference regimes and spectral efficiencies of the interferers. This leads to an updated interference classification, that can be exploited in the matching problem. With this new interference classification, we can define the optimal AP-UE assignments, interference regimes and spectral efficiencies to be used in any 2-users Gaussian Interference Channel for any pair of interferers matched together: the objective now only consists of finding the optimal one-to-one matching of the $2N$ unassigned interferers, that maximizes the total spectral efficiency after interference programming. The problem can be optimally solved by considering a graph-theoretic approach, . Graph theory is a mathematical tool that models pairwise relations between entities through the use of particular mathematical structures known as graphs [101]. Graphs are made of vertices, also known as nodes, and lines connecting them, known as edges. In our optimization problem, the objective consists of finding the optimal disjoint weighted edges matching in a $2N$ complete graph [94, 102, 103], where each edge weight corresponds to the total spectral efficiency after interference processing that could be obtained if the two interferers, represented by the two nodes, were to be coupled together.

The extension of the problem to $M > 2$ AP is also investigated in Section 6.4 and exploits the proposed suboptimal interference classification in M-users Gaussian Interference Channels, detailed in Section 6.4.2. The twofold matching (AP-UE assignments and the matching of interferers) can be investigated as a Non-Linear Programming (NLP) problem [104, 105]. Similar to Integer Linear Programming (ILP) [106], the non-linearity is due to the objective function, as it consists of the spectral efficiency obtained after interference processing. Solving a Non-Linear Programming problem is known to be NP-Hard [107]. For this reason, several suboptimal heuristic algorithms have been developed, such as the Genetic Algorithms (GA). Genetic algorithms [108, 109, 110] are a class of heuristics based on the concept of evolutionary computing that aim at finding the maximum of multi-variable objective possibly non-linear functions, through mechanisms that mimics the natural selection of genes. Introduced in the field of artificial intelligence, GAs are a class of fast converging algorithms that performs particularly well in cases in which the solution must be chosen from a large set. The basic idea behind GAs is to create a set of genetic codes, typically binary strings representing one of the possible elements of the domain of objective functions, and then selecting them through the mechanisms of selection, variation and inheritance. We detail in Section 6.4.3 the proposed Genetic Algorithm that is used to compute a suboptimal solution to our optimization problem. Even though GAs have been implemented to configure several parameters in CRs [111], these algorithms require for each radio to have a vast knowledge on the other radios behavioral rules and possible configurations.

1.2 Thesis Outline

The thesis is composed of two parts, which both include two chapters:

- **Part 1:** Power Efficiency in proactive delay-tolerant networks.

  Chapter 3: In the first half of the thesis, we investigate delay-tolerant proactive networks. In particular we investigate how the system might exploit the offered future knowledge and benefits from it, in terms of energy efficiency. In Chapter 3, we propose a preliminary example, which consists of a single user proactive delay-tolerant problem. It allows to define the key concepts to be used during the first half of this chapter: future knowledge, delay tolerance, and optimization. In Section 3.3, we define the concept of future knowledge and investigate how it can be exploited in delay-tolerant optimization problems. We also investigate the benefits offered by several kinds of future knowledge, in delay-tolerant networks, with scenarios of future knowledge ranging from perfect prior knowledge to zero future knowledge, including scenarios of incomplete/statistical future knowledge. The future knowledge scenarios investigated in the chapter are introduced in Section 3.4. Finally, numerical simulations in Section 3.5 provide interesting insights about how the system benefits from each scenario of future knowledge. The potential performance gain between the optimal perfect knowledge strategy (obtained when perfect knowledge is given) and the worst case scenario (obtained when no knowledge of the future is available) appears to be significant: it then makes sense to look forward to acquiring and exploiting some elements of future knowledge. We also show in Section 3.5.4 that the performance gap depends on the time variations of the channel realizations. More specifically, the performance gains depend on the capability of the system to discern good channel realizations from bad channels realizations and exploit them properly, as a time water-filling scheduler would. Since acquiring a perfect knowledge at any time seems unrealistic (though ideal), we investigate partial and statistical future knowledge schedulers. It turns out that a good statistical knowledge might be sufficient as it allows to approach remarkably the optimal performance bound. Also, acquiring a short-term knowledge, which is realistic, can also enhance the performance of the system.

  Chapter 4: In the next chapter, Chapter 4, we investigate the extension of the previous proactive delay-tolerant problem, to a multi-user scenario.

The conducted analysis, detailed in Section 4.2, reveals that the problem can be modeled as a multi-user non-cooperative stochastic game, which is hard to solve when the number of users becomes large. We detail the reasons of the inherent mathematical complexity and remind the reader about the classical approach used for tackling the complexity (simplifications on the system model and heuristics). We then investigate, in Section 4.5, how the recent advances in Mean field Games theory can be used to simplify the initial problem, by turning it into an equivalent Mean Field Game, with lower complexity. A procedure for analyzing the equilibrium of the Mean Field Game is proposed and simulation results are provided for several channel scenarios: these results highlight significant potential performance gains, in terms of energy efficiency, offered by proactive delay-tolerant methods, capable of exploiting both the offered latency and future context knowledge. The numerical simulation reveals significant performance gains in terms of energy efficiency, offered to the system, when it is able to exploit the latency and the future knowledge.

- **Part 2:** Interference classification, interferers matching and virtual handover.

  Chapter 5: In the second half of the thesis, we investigate the dual problem of the previous optimization problem, i.e. investigate a spectral efficiency optimization under the constraint of a constant short-term power. This considered approach allows to simplify the optimization analysis, as it directly tackles the inherent complexity of power adaptation, that we referred to as 'ping pong effect'. More specifically, we investigate in this chapter, how the recent works on interference classification could be exploited in Radio Resource Management problems, to enhance the total spectral efficiency of the system. In Chapter 5, we present a short tutorial on the concept of interference classification. We investigate in Section 5.3 the optimal interference regimes selection in 2-users Gaussian Interference Channels, in orde to maximize the total spectral efficiency after interference processing. In Section 5.4, we then enhance the initial interference classification problem, by considering coalition of interferers assigned to each AP and consider the matching problem, with the objective of finding the optimal matching of interferers, which maximizes the total spectral efficiency after interference processing of the system. The extension of the previous matching problem is investigated in Section 5.4.

We rapidly detail the reasons why interference classification in M-users Gaussian Interference Channels is hard to investigate and still an open question in literature. Even though it becomes NP-Hard when the number of AP and coalitions M becomes greater than 2, the matching problem can still be investigated without interference classification: we propose a suboptimal genetic algorithm, capable of solving the matching problem. Numerical simulations are then provided and illustrate the potential significant gains offered by both concepts of interference classification and interferers matching.

Chapter 6: In Chapter 6, we investigate the concept of 'virtual handover': when interference classification is considered, the system must reconsider the way it assigns its UEs to APs, as the optimal AP is not necessarily the one providing the best SNR anymore, as proven in Section 6.2.2. We then start again the study of the previous RRM optimization problem considered in the previous chapter, by considering that the system may now decide how to assign its UEs to APs. The objective is threefold: find the AP-UE assignments, find the matching of interferers, and find the interference regimes and spectral efficiencies to be used, so that the total spectral efficiency after interference processing is maximized. We first update, in Section 6.2.3, the interference classification algorithm used in 2-users Gaussian Interference Channels, when the possibility of reassigning UEs to APs is considered. The derived classifier can then be reused in the matching problem, with $M = 2$ APs and $2N$ unassigned interferers, as detailed in Section 6.3. The matching problem is then optimally solved using the Edmonds algorithm, from Graph Theory. The extension of the problem to $M > 2$ APs is then considered and the two main issues are addressed. First, we propose a suboptimal game-theoretic approach to interference classification in M-users Gaussian Interference Channels, in Section 6.4.2. Then, we investigate the remaining twofold matching, which consists of matching interferers and assigning UEs to APs, assuming they will implement interference classification, according to the game-theoretic approach detailed in Section 6.4.2. The twofold matching can then be modeled as a Non-Linear Programming problem, which is known to be NP-Hard: we then propose a suboptimal Genetic Algorithm in Section 6.4.3, which is used for solving the problem. Numerical simulations provide interesting insights on the potential gains offered by the three concepts on which the threefold optimization relies on, namely: interference classification, interferers matching and virtual handover.

1.3 Publications

The work in this thesis has been summarized and presented in the following contributions:

Journal Papers

- **J1:** [112] De Mari, M. and Calvanese Strinati, E. and Debbah, M., '5G Network Performance Optimization basedon Matching and Interference Classification', *submitted IEEE Transaction on Wireless Communications, 2015.*

- **J1:** [113] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Mean Field Games Power Control for Proactive Delay-Tolerant Networks', *to be submitted to IEEE Transactions on Wireless Communications, 2015.*

Conference Papers

- **C1:** [114] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Concurrent data transmissions in green wireless networks: When best send one's packets?', *9th International Symposium on Wireless Communications Systems (ISWCS), 2012.*

- **C2:** [115] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Energy-efficiency and future knowledge tradeoff in small cells prediction-based strategies', *12th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), 2014.*

- **C3:** [116] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Two-regimes interference classifier: An interference-aware resource allocation algorithm', *IEEE Wireless Communications and Networking Conference (WCNC), 2014.*

- **C4:** [117] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Matching Coalitions for Interference Classification in Large Heterogeneous Networks', *IEEE 25th Annual International Symposium on Personnal Indoor and Mobile Radio Communications, 2014.*

- **C5:** [118] De Mari, Z. Becvar, M. and Calvanese Strinati, E. and Debbah, M., 'Interference Empowered 5G networks', *5GU Conference, 2014*.

- **C6:** [119] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Mean Field Games Power Control for Proactive Delay-Tolerant Networks', *to be submitted, 2015.*

- **C7:** [120] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'A Game Theoretical Approach to Interference Classification in M-Users Gaussian Interference Channels', *to be submitted, 2015.*

Patents

- **P1:** [121] E. Calvanese Strinati, M. De Mari, M. Debbah, 'PROCEDE DE PRISE DE DECISION D'UN TRANSFERT DE COMMUNICATION DANS UN CONTEXTE INTERFERENTIEL INTRA-BANDE', Filed in February 2015.

Note: All the papers, excepted the patent, are available online on my personal website at `http://matthieu-de-mari.fr/publications-and-papers/`.

Chapter 2

Synopsis en Francais

2.1 Contexte de Recherche et Motivations pour les Réseaux Sans Fil Green

2.1.1 Tendances actuelles

Du fait de la croissance exponentielle du trafic de communications, le secteur industriel des technologies de l'information représente actuellement 2% des émissions de carbone et ce chiffre est supposé doubler d'ici à 2020. Cette augmentation est liée au fait que les utilisateurs, toujours plus nombreux, recherchent une qualité de service toujours plus importante [1]. Une contribution de 2% peut sembler dérisoire, mais elle représente en réalité près de 250 à 300 millions de tonnes de carbone émises et rejetées dans l'atmosphère, comme stipulé par une récente étude de Greentouch [2]. En plus des sujets environnementaux, les raisons qui poussent aujourd'hui les opérateurs mobiles à réduire la consommation énergétique du réseau font écho à un second problème d'ordre économique: la facture énergétique de Vodafone, en 2007-2008, représentait près de 3000GWh, ce qui correspondait à 1.45 millions de tonnes de carbone et représentait un coût de plusieurs centaines de millions de dollars [3]. Ce coÃ»t énergétique représente par ailleurs une partie non négligeable des dépenses engagées par un opérateur: on l'estime aux environs de 18% sur le marché européen 18% [3]. Dans le même temps, les revenus des opérateurs, eux, ne croient que faiblement à un taux inférieur à 10% par an [1]. Pour cette raison, il apparait nécessaire pour les opérateurs aujourd'hui de réduire les coûts énergétiques à tout prix. Dans un tel contexte, de nombreuses recherches ont été menées autours de concepts de réseaux télécom "‘green"’, ou autrement dit énergétiquement optimisés et efficaces [4]. L’objectif fixé pour ces réseaux est ambitieux, puisqu’il ne s’agit ni plus ni moins, que de multiplier par 1000 l’efficacité énergétique du réseau d’ici à 2020, comparé à sa valeur de 2010 [4, 5]. Ce challenge est en réalité loin d’être trivial, puisqu’il faut dans le même temps s’adapter à la demande toujours plus importante de qualité de service des utilisateurs. Plusieurs pistes possibles ont ainsi été identifiées pour atteindre cet objectif. Puisque la majorité du coÃ»t de puissance vient en réalité des points d’accès (80 %), il apparait que la piste la plus évidente consiste à rendre ces points d’accès énergétiquement plus efficaces, non seulement dans leurs modes de fonctionnement, mais aussi dans leur déploiement [6]. Parmi les pistes possibles d’améliorations, trois grandes familles se dégagent:

- Les techniques de gestions de ressources, énergétiquement efficaces, avec entre autres des techniques d’allocation de ressources, de contrôle de puissance, des modes veille pour les points d’accès, etc.

- L’amélioration du déploiement des point d’accès, via la densification et l’hétérogénéisation du réseau.

- Des techniques de traitement multi antennes, en particulier avec des grands nombre d’antennes (Massive MIMO), des techniques de beamforming ou de spatial multiplexing.

Dans cette thèse, nous nous focalisons principalement sur des techniques issues des deux premières familles exclusivement.

2.1.2 Pistes de Recherche pour les réseaux "‘green"’ - Gestion de ressources, radios cognitives et contrôle de puissance

2.1.2.1 Controle de puissance et mode veille

Un premier ensemble de solutions suggère qu’il est possible d’améliorer l’efficacité du réseau au moyen de techniques dites de gestion de ressources, incluant en particulier du contrôle de puissance. Dans de telles techniques, rendues possibles par l’émergence des concepts de radios cognitives [7], l’objectif consiste à adapter la puissance d’émission utilisée au contexte de transmission du réseau, afin de maximiser une fonction d'objectif. Dans les travaux d'optimisation énergétique, cette fonction est classiquement liée à la consommation énergétique du réseau. Parmi tous les paramètres de contrôle pouvant être adaptés, la puissance de transmission apparait souvent comme étant le candidat à la fois le plus évident, mais aussi le plus approprié, et ce pour deux raisons. Tout d'abord, et comme mentionné précédemment, il peut être observé que la majeure partie du coÃ»t énergétique des points d'accès est lié à la puissance de transmission [8], et que la consommation de puissance sur ces points d'accès affecte directement l'efficacité globale du réseau complet. De deux, l'adaptation de puissance est en réalité directement liée à l'interférence observée entre les points d'accès, et elle est souvent la cause de bien des soucis lorsqu'il s'agit de transmettre. Pour ces raisons, de nombreux travaux on récemment proposé d'optimiser les puissances de transmission utilisées, afin d'améliorer l'efficacité énergétique du réseau, tout en garantissant une qualité de service minimale aux utilisateurs [9, 10]. Parmi les résultats observés, on observe un phénomène de gagnant-gagnant, puisque cette adaptation de puissance, permet non seulement de réduire la consommation énergétique, mais permet aussi en réalité de contenir l'interférence, ce qui en conséquence améliore l'efficacité du réseau: on peut donc faire mieux pour moins cher [11, 9, 10]. Parmi les travaux fondamentaux sur ce sujet, l'optimisation décrite par Kandukuri et Boyd fait aujourd'hui référence [12].

Dans la réalité, la consommation de puissance au niveau d'un point d'accès peut se décomposer en deux parts. La première partie, relativement constante, prend en compte le coût énergétique de fonctionnement du point d'accès, avec entre autres le refroidissement, les processeurs, etc. Une deuxième partie du coût est dynamique et dépend de la charge de travail associée à ce point d'accès, ainsi que des puissances de transmissions utilisées [8]. Sur les points d'accès de type macro, il a en particulier été observé que le coût statique représentait la part la plus importante. Il apparait donc intéressant de pouvoir identifier les moments où le point d'accès n'est pas utilisé, afin de l'éteindre et de sauver ainsi énormément d'énergie [13, 14, 15]. Des optimisations ont ainsi été menées dans ce sens: dans ce papier [16] par exemple, les auteurs proposent d'établir un planning de transmission afin de maximiser le temps pendant lequel le point d'accès peut être éteint.

2.1.2.2 Tolérance au délai, prédictions et efficacité énergétique

Dans le même temps, de récents travaux ont indiqué que la majorité des transmissions actuelles étaient en réalité non urgentes: mises à jour d'applications et d'autres échanges de data n'ont pas nécessairement besoin d'être réalisé instantanément, au contraire il est souvent même préférable d'attendre d'être dans un contexte de transmission peut coûteux pour transmettre. Un utilisateur préférera sans doute ainsi procéder à la mise à jour de ses applications chez lui via son wifi, plutôt que dans le métro. De cette idée est apparu le concept de réseau tolérant à la latence ou au délai[17, 18]. Dans ce contexte de transmission, on choisit alors volontairement de laisser le réseau transmettre quand il le souhaite, le laissant ainsi adapter librement ses puissances de transmissions pour réduire sa consommation globale, en lui imposant simplement un délai maximal. il s'agit d'un tradeoff classique dans la littérature [19] et il amène directement aux concepts de réseaux tolérants à la latence [20, 21, 22].

Dans [23, 24, 25], les auteurs se posent la question de savoir quand le système doit sortir de veille pour transmettre, afin de minimiser la consommation de puissance tout en respectant la contrainte de deadline imposée. Il a également été observé que ces techniques pouvaient également permettre de faciliter les problèmes de congestion observés dans le réseau [20, 26, 27]: les réseaux peut ainsi librement s'adapter et réaliser les transmissions non-urgentes dans des moments où les ressources sont disponibles en quantité, à faible coût.

Dans le même temps, il a également été observé que l'utilisation du réseau et le comportement de ses utilisateurs était très facilement prédictible [28, 29]. La mobilité d'un utilisateur peut par exemple être facilement prédite, car elle est définie par des routes, des trottoirs et des chemins classiques: il est donc facile de prévoir la trajectoire à court terme sur la base de son déplacement actuel [30, 31]. En couplant cette prédiction de mobilité à des cartes radio, donnant la qualité de lien moyenne observée pour n'importe quelle position géographique peut ainsi permettre d'obtenir une prédiction de la qualité de lien future des utilisateurs du réseau [32, 33, 34]. En conséquence, des travaux ont donc cherché à intégrer ces modèles prédictifs dans des approches d'optimisation tolérantes à la latence, ce qui amène à ce que l'on appelle classiquement dans la littérature 'des réseaux tolérants à la latence proactifs' [35, 36, 37]. Des gains de diversité ont par exemple été analytiquement démontrés. Dans ces papiers [38, 39, 40], le système peut définir des prédictions sur les contextes de transmissions des utilisateurs, ainsi que sur les requêtes futures des utilisateurs:

le système peut alors librement s'adapter à un futur prévisible et réduire sa consommation globale.

2.1.3 Un premier exemple illustratif mono-utilisateur

Dans la première partie de cette thèse, nous nous intéressons aux méthodes qui permettent au système de pouvoir adapter ses paramètres de transmissions, afin de maximiser une fonction d'objectif, tout en prenant en compte non seulement des contraintes de temps et de performance, mais également des prédictions sur le futur contexte de transmission du réseau. Nous décrivons ainsi un premier exemple illustratif dans le Chapitre 3, pour un modèle similaire à [41, 42]. Nous considérons alors un système composé d'un point d'accès dont l'objectif est de réaliser une transmission d'un paquet imposé à moindre coût, avant une deadline imposée. Le système peut librement décider quand transmettre et à quelle puissance. La taille du paquet restant à transmettre décroit à chaque time slot, en fonction du SINR, qui est lui même une fonction de la puissance (qui peut être adaptée) et de la qualité du lien (sur laquelle le système peut formuler une prédiction). En pratique, définir la stratégie optimale de transmission à chaque time slot est purement équivalent à un problème de contrôle de puissance, reposant sur de l'optimisation convexe sous contraintes [43]. Dans l'approche classique, lorsque le futur est parfaitement connu, le problème est résolu au moyen d'un lagrangien, associé au système, démarche classiquement appelée Karush-Kuhn-Tucker(KKT) [44]. Cette démarche permet ainsi de réécrire le problème via un ensemble conditions équivalentes, auxquelles la stratégie optimale répond, permettant ainsi de la calculer. Une seconde approche, capable de prendre en compte des incertitudes sur le futur, repose sur de la programmation dynamique, où le système adapte sa stratégie instantanée à l'espérance du futur [45].

Lorsque le système a une connaissance parfaite du futur, la solution optimale est connue et peut être classiquement calculée via un algorithme de 'time water-filling' qui découle directement des conditions de Karesh-Kuhn-Tucker [44]. Les techniques de water-filling pour le contrôle de puissance ont été très largement détaillées dans la littérature, en particulier dans les papiers suivants [43, 46, 47, 48, 49, 50]. Cependant, obtenir une connaissance parfaite du futur est malheureusement très peu réaliste, en pratique les modèles de prédictions doivent prendre en compte l'incertitude de mesure et de prédiction.

Dans ce premier chapitre, nous nous intéressons donc à divers degrés de connaissance du future, plus ou moins idéaux, afin de voir comment le système en tire un bénéfice. Cette démarche permet ainsi de mettre en évidence les outils mathématiques utilisés, mais aussi d'identifier les éléments utiles pour le système en termes de prédiction. Les scenarios considérés dans ce chapitre sont:

- **la connaissance parfaite du future:** le système a une connaissance parfaite du futur en tout instant. Ce scénario, idéal, permet de définir la performance maximale du système, via du time water-filling.
- **l'absence totale de connaissance du futur:** elle constitue l'approche état de l'art et permet de définir le mieux qu'il soit possible de faire, lorsqu'aucune connaissance du futur n'est disponible. La stratégie optimale peut être obtenue via des conditions KKT.
- **connaissance statistique:** le système connait les statistiques exactes du canal mais ne connait les réalisation exactes du canal qu'au début de chaque time slot. Le problème est alors classiquement résolu via backward programming.
- **connaissance exacte à court terme exclusivement:** dans cette configuration, le système connait exactement le futur proche, mais ne peut formuler aucune prédiction à long terme. Une approche backward programming est considérée.
- **connaissance exacte à court terme et statistique à long terme:** il s'agit du scénario précédent, dans lequel la connaissance à long terme du futur est maintenant donnée, comme étant les statistiques du canal. Là encore, une approche backward programming est considerée.

Ces scenarios sont explicitement détaillés dans la Section 3.4. Dans chaque scénario, nous détaillons théoriquement la démarche permettant le calcul de la stratégie optimal, puis des simulations numériques permettent de mettre en évidence les gains théoriques offerts par divers degrés de connaissance du futur.

Ce premier exemple illustratif permet de mettre en évidence des questions fondamentales et d'y apporter des débuts de réponses:

- **Comment le système peut-il exploiter une telle connaissance du futur?** L'approche proposée repose sur les réseaux tolérants à la latence. L'optimisation mathématique sous contrainte est capable de prendre en compte cette connaissance du futur et permet de définir des stratégies de puissance efficaces énergétiquement.

- **Cette connaissance du futur peut-elle apporter un gain?** Les simulations numériques démontrent un gain de performance, observé entre le scenario de connaissance parfaite et l'absence totale de connaissance. Mettre en évidence un gain important était nécessaire, pour deux raisons. Tout d'abord, nous nous attendons à ce que la performance des scénarios de connaissance imparfaite du futur aient des performance situées entre ces deux scénarios, préférablement proche du cas de connaissance parfaite. De deux, il est nécessaire d'avoir un gain massif, dépassant le coût de learning, à savoir le coût à payer pour accéder à une telle connaissance du futur. Nous ne détaillons pas dans ce manuscrit ce coût de learning mais apportons des éléments de réponse dans la Section 3.6.

- **Quel type de connaissance est réellement utile au système?** L'analyse révèle que le scénario de connaissance statistique permet d'approcher notablement la performance obtenue dans un scénario de connaissance idéale. Avoir une connaissance idéale à court terme peut sembler de faible intérêt, mais en réalité, elle peut s'avérer suffisante à apporter des gains eux aussi significatifs.

2.1.4 Réseaux tolérants à la latence proactifs en contexte multi-utilisateurs: jeux stochastiques

Dans le second chapitre de la première moitié de ce manuscrit (Chapitre 4), nous investiguons l'extension du problème précédent dans un contexte multi-utilisateurs. Dans ce scénario, nous considérons $N \geq 2$ paires de point d'accès et d'utilisateurs. Là encore, les points d'accès doivent réaliser la transmission d'un paquet imposé à leurs utilisateurs respectifs (ces tailles peuvent varier d'un utilisateur à l'autre) et ce avant une deadline commune. Les points d'accès peuvent à nouveau adapter leurs puissances de transmission, et cherchent à minimiser leurs consommations respectives. Le problème se complexifie énormément ici, puisqu'il est désormais nécessaire de prendre en compte l'interférence entre les différents points d'accès. La vitesse de décroissance du paquet d'un utilisateur dépend maintenant du SINR observé à chaque récepteur, et celui-ci prend désormais en compte l'interférence, résultant des contributions et actions de l'ensemble des points d'accès. Nous considérons ici que la prédiction du futur donnée à chaque point d'accès suit une loi de Itô, ce qui constitue un modèle classique de prédiction [51]. Dans un tel contexte, le calcul des stratégies optimales peut être obtenus via une analyse de jeu stochastique [52, 53]. Ces jeux ont classiquement été utilisés dans le domaine de la finance pour modéliser des interactions et compétitions entre des utilisateurs souhaitant accomplir la même tâche, de façon rationnelle, mais de façon non-coopérative [53]. Dans ces jeux coopératifs, la stratégie optimale est obtenue via un équilibre de Nash [54, 53], état du système dans lequel chaque paire recoit une stratégie qu'elle juge indépendamment optimale: aucun d'entre eux ne peut espérer un gain plus important en déviant indépendamment de la stratégie qui lui a été assignée dans l'équilibre de Nash. On obtient ce faisant, une configuration stable pour le système.

Deux approches sont classiquement considérées pour procéder à la définition de l'équilibre de Nash d'un jeu stochastique:

- L'équilibre peut être caractérisé par N équations aux dérivées partielles, couplées, nommées équations de Hamilton-Jacobi-Bellman, comme détaillé dans [53]. Résoudre ce système à équations couplées devient très rapidement problématique lorsque les dimensions du système, et en particulier le nombre d'éléments dans le système N devient trop important. Cette complexité vient du fait qu'il est nécessaire de prendre de nombreux paramètres en compte parmi lesquels les différents canaux entre tous les points d'accès et tous les utilisateurs, mais aussi les paquets à transmettre de tous les utilisateurs.

- Quand il n'y a pas d'incertitude, et donc pas de partie stochastique dans l'analyse, une approche de time water-filling itérative peut être considérée afin d'approcher l'équilibre de Nash [55, 56, 57, 58, 59]. Dans cette méthode, chaque utilisateur adapte tour à tour sa stratégie de puissance, jusqu'à atteindre un point de convergence. Lorsqu'un point d'accès adapte sa stratégie, il redéfinit en réalité l'interférence perçue par les autres utilisateurs, qui ne sont alors plus satisfaits de leur stratégie de puissance précédente et souhaitent à nouveau l'adapter. L'intégralité de l'algorithme repose sur cet effet de ping-pong [55]. Il doit cependant être noté que cet algorithme a un temps de convergence qui dépend très fortement des dimensions du système N, qui doivent rester petites afin de garantir une convergence en des temps raisonnables [60].

Ces deux approches semblent donc poser problème lorsque le nombre d'utilisateurs N devient trop grand. Ceci est problématique, en particulier dans un contexte qui tend à densifier le réseau. Des solutions pour contourner ce problème de complexité mathématique existent cependant:

- Dans des scénarios où les canaux restaient constants au cours du temps, il était possible de calculer les stratégies optimales via un système de N équations linéaires couplées [61].
- Des approches heuristiques, sous-optimales peuvent également être considérées.

2.1.5 Jeux à champs moyens

Les multiples interactions entre utilisateurs semblent ici poser problème et amènent une complexité mathématique plutôt dérangeante. Cependant, il peut être observé trois choses. Tout d'abord, par définition de l'interférence, il peut être observé que l'impact d'un utilisateur seul, lorsque le système devient de plus en plus grand, devient lui de plus en plus faible. Par ailleurs, la perception de l'interférence aux récepteurs ne prend pas en compte les contributions individuelles des compétiteurs séparément, mais au contraire, ce terme d'interférence repose ni plus ni moins sur la somme des contributions de tous les utilisateurs compétiteurs. Enfin, notre modèle présente de nombreuses symétries entre les utilisateurs: ils ont globalement le tous un objectifs similaire à un choix de paramètre près, et ont des objectifs et comportements équivalents face au problème d'optimisation commun qui leur est imposé. En pratique, il est possible d'exploiter ces propriétés de symétries et ce modèle d'interaction entre utilisateurs modélisé par le terme d'interférence, afin de contourner la complexité mathématique liée à un grand nombre d'utilisateurs [62]. La théorie des jeux à champs moyens repose sur cette idée et permet de transiter d'un problème à N utilisateurs, qu'on ne sait pas facilement résoudre, vers un jeu équivalent, dit à champ moyen [63, 64, 65]. Ce jeu équivalent, à champ moyen, présente l'avantage d'avoir une complexité réduite, celle d'un problème à deux corps, là où le problème initial en comptait N.

Quelques papiers ont récemment proposé d'exploiter cette théorie mathématique pour relancer l'analyse de problèmes non-coopératifs stochastiques, jusque-là laissés de côté, du fait de leurs complexités mathématiques. En particulier, dans [66, 67], il est investigué un problème d'optimisation énergétique sous contrainte d'un SINR minimal garanti à chaque utilisateur, à un instantané donné. Dans [68], une analyse similaire est menée dans un contexte smart grid, avec des véhicules électriques. Dans [69, 61], les joueurs sont des transmetteurs, qui peuvent adapter leurs puissance de transmissions à plusieurs éléments de contexte du réseau (qualité du lien, batterie, etc.), l'objectif est là encore de minimiser la consommation énergétique, tout en garantissant une qualité de service minimale à tous. De la même façon, nous proposons d'exploiter ces récentes avancées dans le domaine de la théorie des jeux à champ moyen, pour débloquer l'analyse de notre problème précédent. Nous détaillons ainsi dans le chapitre 4, l'analyse du problème dans sa version stochastique non-coopérative, puis détaillons la transition vers un jeu à champ moyen. L'analyse de ce jeu à champ moyen est ensuite détaillée et les équations permettant d'en caractériser son équilibre sont définies. Pour différents scénarios d'évolution du canal et divers modèles de connaissance du futur, nous définissons les stratégies optimales utilisées par chaque utilisateur du système. La performance de ces stratégies optimales sont finalement comparées dans le cadre de simulations numériques, et permettent alors de mettre en évidence un gain significatif entre la stratégie optimale d'une part et des stratégies état de l'art, incapables de prendre en compte la latence et/ou une éventuelle connaissance du futur offerte au système.

Dans des scénarios où le canal ne varie pas, les stratégies de transmissions optimales peuvent être explicitement calculées, via une méthode KKT et même via une heuristique classique ne nécessitent pas une connaissance du futur. On observe alors que la stratégie obtenue par le jeu à champ moyen est très fortement similaire à la stratégie optimale calculée via KKT, ce qui semble confirmer la garantie d'optimalité de l'approche jeu à champ moyen. Lorsque le canal varie au cours du temps, l'approche KKT n'est plus utilisable et il n'est plus possible de calculer simplement l'optimum, hormis via une approximation de jeu à champ moyen. La stratégie obtenue par approche jeu à champ moyen possède cependant un avantage de taille, à savoir qu'elle est capable d'exploiter non seulement la latence offerte au système, mais également une connaissance du futur. C'est la raison pour laquelle elle performe mieux que les autres stratégies état de l'art utilisées comme référence dans notre analyse. Deux gains cumulables apparaissent alors: un premier gain est lié à la capacité du système à prendre en compte la latence qui lui est offerte pour transmettre, et ce gain est exacerbé par la connaissance du futur donnée au système, ce qui lui permet d'identifier à l'avance les bons contextes de transmissions, pour transmettre efficacement, à faible coût.

2.1.6 Amélioration du déploiement du réseau, en vue d'une plus grand efficacité énergétique

2.1.6.1 Vers des réseaux hétérogènes à larges dimensions

Une deuxième approche aujourd'hui largement considérée pour permettre l'amélioration de l'efficacité énergétique des réseaux mobiles consiste en la densification et l'hétérogénéisation des réseaux [70, 71, 72]. Cette solution permet notamment d'améliorer la performance du système en réduisant la taille moyenne des cellules, améliorant ainsi la réutilisation spectrale des ressources [73]. La réduction des distances permet également de pouvoir non seulement transmettre à de plus faibles puissances, mais permet également de transmettre avec des meilleurs SINR, ce qui résulte au final en une meilleure performance pour le système [72]. C'est donc aujourd'hui une solution gagnant-gagnant pour les opérateurs mobiles. Cette solution est cependant plus difficile à gérer pour les opérateurs, du fait de leur plus grand nombre d'éléments, ainsi que du fait de la diversité d'éléments présents dans le réseau (pico/femto/macro cellules, relais, etc.). Par voie de conséquence, la gestion de l'interférence devient elle aussi plus complexe. A tel point que l'interférence puisse aujourd'hui être considérée comme un problème majeur.

2.1.6.2 L'Interférence, l'Ennemi Universel

Dans l'approche classique, l'interférence est perçue comme un ennemi, qui compromet les transmissions. Le traitement classique, consiste à assimiler l'interférence à une source additionnelle de bruit. Pour cette raison, et afin de garantir une efficacité spectrale suffisante, il apparait comme nécessaire d'éviter l'interférence autant que possible, et dans les cas où elle ne peut être évitée, de s'assurer qu'elle restera à un niveau suffisamment faible pour ne pas compromettre les transmissions. Pour cette raison, les techniques permettant la gestion de l'interférence peuvent être classifiées en deux grandes familles : celles qui permettent d'éviter l'interférence et celle qui visent à en limiter les effets de dégradation.

La façon la plus classique d'empêcher l'interférence, entre des sources cherchant à transmettre dans une même région géographique, consiste à forcer l'orthogonalisation des transmissions, à savoir forcer les différentes sources en compétition à transmettre sur des ressources spectrales différentes. Parmi les techniques permettant classiquement de réaliser l'orthogonalisation on peut notamment noter : L'orthogonalisation via temps/fréquence (TDD/FDD et

TDMA/FDMA), ainsi que les approches Orthogonal Frequency Division Multiple Access (OFDMA) et Code/Space Division Multiple Access (CDMA/SDMA) [74, 75]. Cependant, il faut noter que les ressources disponibles sont souvent en très faibles quantités et à ce titre ne peuvent pas être efficacement partagées entre les nombreux éléments du réseau. Par ailleurs, ces techniques ont une faible réutilisation spectrales, et l'on sait aujourd'hui qu'elles sont faiblement performantes. Une seconde approche consiste également à envisager de limiter les interférence entre cellules voisines en s'assurant qu'elles ne transmettront pas sur des ressources communes: une telle allocation de ressource s'avère être en réalité un problème de graph coloring [76, 77, 78].

Un second ensemble de techniques cherche généralement à contenir l'interférence générée au sein du réseau, afin qu'elle subsiste à des niveaux faibles et acceptables. Les approches classiques de ce genre, sont souvent illustrées avec du contrôle de puissance où l'objectif consiste à adapter les puissance des différents transmetteurs du système, afin de contenir l'interférence générée. Ce genre d'approches est typiquement détaillée dans la première moitié de ce manuscrit et a également été passé en revue dans la littérature [79, 10, 12, 11]. Il doit cependant être noté que ce genre de techniques amène souvent des problèmes mathématiques extrêmement complexes à gérer, en particulier lorsque les dimensions du système deviennent larges, comme mentionné précédemment et dans les papiers suivants [80, 81, 60].

2.1.6.3 L'interférence: Amie ou Ennemie ?

Les méthodes mentionnées précédemment ont en commun qu'elles partent toutes du principe que l'interférence doit être assimilée à une source de bruit complémentaire. Ces méthodes oublient cependant de prendre en compte les récentes avancées dans le domaine du traitement de l'information, en particulier les nouveautés relatives au traitement de l'interférence, dans des contextes où l'interférence s'avère être forte. En pratique, Carleial [82] et plus tard, Han & Kobayashi [83] ont montré qu'il était parfois possible d'exploiter les propriétés intrinsèques de l'interférence, afin d'obtenir des performances notables. L'astuce permettant de contourner le problème repose ici sur le fait que l'interférence n'est pas seulement du bruit, mais bien issue d'un autre signal. Sur la base de ce constat, Carleial suggère que lorsque l'interférence est forte, ce qui pose problème n'est pas réellement l'interférence, mais bel et bien la technique permettant de gérer cette interférence. Il proposa ainsi une technique, connue aujourd'hui sous le nom de Successive Interference Cancellation (SIC), dont l'objectif consiste à simplement décoder l'interférence en premier lieu et en présence du signal primaire (alors traité comme une source de bruit), puis de soustraire le signal interférent fraichement décodée du signal reçu: le signal obtenu est alors vide de toute interférence [82]. Cette technique s'avère aujourd'hui être optimale dans les scénarios à forte interférence. On peut alors conclure de cet exemple qu'une unique technique de gestion de l'interférence s'avère insuffisante et qu'il faut au contraire envisager d'adapter le traitement de l'interférence au contexte d'interférence.

Cette observation a aujourd'hui été admise, dans le design du futur réseau 5G [84]. Elle a aussi inspiré les travaux de Etkin & Tse [85], qui ont cherché à identifier les meilleures techniques de traitement d'interférence à utiliser en toute circonstances d'interférence. Ils ont identifié 5 techniques majeures et défini les configuration de SNR/INR dans lesquelles chacune de ces techniques s'avère être en réalité la plus adaptée. Ils ont ainsi défini des régions d'interférence, pour lesquelles chacune des 5 techniques s'avère être la meilleure, ainsi que les performances obtenues après traitement de l'interférence. Les régions SNR/INR sont définies au moyen d'un indicateur α, défini comme $\alpha = \frac{log(INR)}{log(SNR)}$. Les diverses régions et performances obtenues après traitement ont amenée à la réalisation d'une figure en forme de W, célèbre et représentée en Figure 2.1.

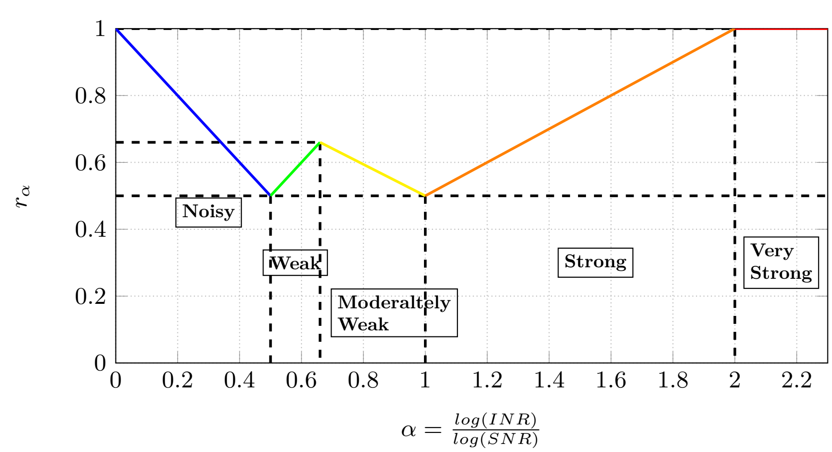

Figure 2.1: Generalized degrees of freedom, according to the α value. This 'W-shaped' curve exhibits an interference classification into 5 interference regimes.

Cette classification à 5 régimes d'interférence fut finalement réduite à 3 régimes par Abgrall [86, 87]. Dans cette classification, l'interférence ne peut être perçue que de 3 façons : faible (auquel cas elle est traitée comme une source de bruit), forte (auquel cas elle est traitée via SIC) ou entre deux (auquel cas, elle est évitée, via orthogonalisation).Pour justifier cette réduction, Abgrall a suggéré de conserver les techniques fortes et faibles interférence, car elles sont optimales, et a groupé les 3 régimes intermédiaires en un seul, l'orthogonalisation. Les raisons qui ont motivé ce choix reposent sur la facilité d'implémentation en pratique des techniques d'orthogonalisation, là où les techniques mêlant Joint Decoding s'avèrent en réalité n'être encore que trop théoriques. Nous proposons de détailler très rapidement ces différents traitements d'interférence dans un tutoriel décrit dans la Section 5.2.

Dans cette seconde moitié du manuscrit, nous proposons de nous attarder plus spécifiquement sur un problème d'optimisation visant à l'amélioration de la performance du système, via l'adaptation des traitements prodigués à l'interférence, dans un scénario où les puissances de transmissions sont imposées. En ce sens, ce nouveau problème d'optimisation peut sembler être le problème dual du problème d'optimisation décrit dans la première moitié du manuscrit. De la même façon que des récents travaux, nous proposons donc d'exploiter les récentes avancées en classification d'interférence (e.g. [88, 89]) et détaillons notre approche en Chapitre 5. Pour ce faire, nous choisissons de considérer dans une premier temps un système de 2-users Gaussian Interference Channel. L'objectif consiste alors à définir les meilleurs traitements d'interférence à réaliser au niveau de chaque récepteur, afin de maximiser la performance globale du système. Les traitements d'interférence considérés ici, sont ceux de la classification à 3 régimes de Abgrall. De cette façon, nous obtenons à problème d'optimisation à faible complexité, puisque les puissance restent fixées, et que l'on s'épargne ainsi les effets de ping-pong décrits précédemment [60, 86]. L'analyse permet de démontrer que seuls deux régimes subsistent, correspondant aux régimes en faible et forte interférence : il est donc possible de gérer l'interférence de façon efficace, sans pour autant chercher à tout prix à l'éviter, via de l'orthogonalisation. C'est un résultat connu en littérature, mais que nous confirmons aujourd'hui. Cette première étape d'optimisation est réutilisée plus tard dans des problèmes plus complexes d'optimisation, que nous détaillons ci-après.

Nous abordons rapidement en Section 6.4.2, l'extension de l'approche considérée ici, dans le cas des systèmes M-users Gaussian Interference Channel.

Plusieurs papiers, dont [88], ont essayé de s'attaquer au problème de classification d'interférence en contexte multi-utilisateurs. Les techniques de gestion de multi interférence, dites de Group SIC, iterative k-SIC et de k-Joint Decoding doivent alors être considérées parmi les régimes possibles [90], ce qui amène bon nombre de nouveaux régimes à passer en revue. Ces trop nombreux nouveaux régimes, rendent l'analyse au cas par cas longue et fastidieuse, ce qui est la raison pour laquelle le problème de classification multi-utilisateurs et aujourd'hui encore une question ouverte en recherche. D'autres techniques dites d'alignement d'interférence s'avèrent ausi prometteuses et ont permis notamment d'obtenir des performances similaires aux techniques de classification d'interférence en contexte de 2-user Gaussian Interference Channel [91, 92, 93]. Il doit cependant être noté que ces techniques sont purement théoriques à l'heure où nous écrivons ces lignes, ce qui les rend très peu pratiques en implémentation. Pour malgré tout s'attaquer au problème de classification d'interférence en systèmes M-users Gaussian Interference Channels, nous proposons dans cette thèse, une approche théorie des jeux non-coopérative. Bien que sous-optimale, cette approche non seulement nous simplifie grandement la tâche, mais amène à des résultats plutôt notables. Nous détaillons plus spécifiquement cette approche en Section 6.4.2.

2.1.7 Matcher des 'Interféreurs Amis', pour exploiter la Classification d'Interférence

Dans les Sections 5.4 et 5.5, nous étendons le problème précédemment considéré dans un nouveau scénario comprenant des coalitions d'utilisateurs, associés à chaque point d'accès. L'objectif ici consiste à définir des groupes d'interféreurs, transmettant sur les mêmes ressources spectrales. L'objectif ici est d'associer ensemble des 'interféreurs amis', à savoir des interféreurs capables de traiter efficacement leurs interférences réciproques. Il s'agit donc d'un problème de matching 'un pour un', pour lequel une théorie mathématique solide existe [94, 95]. Dans notre cas, nous révélons au cours de notre analyse que notre problème de matching rentre dans un cas particulier de M-Dimensions Multidimensional Assignment Problems (MAP), qui peuvent être résolus de façon optimal avec l'algorithme de Kuhn-Munkres lorsque $M = 2$ [96, 97, 98]. Le problème devient cependant NP-Hard quand $M > 2$ et il faut alors considérer des approches sous-optimales [99].

2.1.8 Classification: Graph Theory, Integer Linear Programming and Genetic Algorithms

Dans le Chapitre 6, nous investiguons la possibilité supplémentaire d'améliorer l'efficacité de la classification d'interférence via la réassignation des utilisateurs aux points d'accès. Lorsque l'interférence était considérée comme une source de bruit, il apparaissait évident que le meilleur point d'accès auquel assigner un utilisateur était celui qui donnait le meilleur SNR, car par voie de conséquence, c'est alors le point d'accès qui donne le meilleur SINR et la meilleure efficacité spectrale après traitement de l'interférence. Cependant, cette idée ne tient plus dès lors que d'autres traitements d'interférence peuvent être considérés [100]. C'est la raison pour laquelle nous investiguons en Chapitre 6, l'extension du problème d'optimisation précédemment détaillé en Chapitre 5, en considérant à présent que le système peut librement réaffecter ses utilisateurs aux points d'accès.

Nous choisissons d'abord de réinvestiguer la classification d'interférence en contexte de 2-users Gaussian Interference Channel, en prenant cette fois en compte la possibilité de réassigner des utilisateurs. Nous obtenons ce faisant une nouvelle classification, que nous pouvons alors exploiter dans le problème de matching, comme fait précédemment. Le problème de matching est simplement résolu via une approche de théorie des graphes[101], puisqu'elle est strictement équivalente à un problème dit de 'optimal disjoint weighted edges matching in a $2N$ complete graph' [94, 102, 103], où les poids assignés à chaque lien correspondent en réalité à l'efficacité spectrale maximale obtenue pour une paire d'interféreurs. Cette efficacité maximale est alors simplement calculée via notre nouvelle classification d'interférence.

L'extension du problème précédent à $M > 2$ points d'accès est également passée en revue dans ce manuscrit, en Section 6.4. Elle repose et exploite la classification d'interférence sous-optimale en système M-users Gaussian Interference Channels, que nous avons détaillé en Section 6.4.2. Le problème de matching à deux degrés (assignations d'utilisateurs aux points d'accès et matching d'interféreurs) repose alors sur un problème de type Non-Linear Programming (NLP) [104, 105]. Bien que similaire à de l'Integer Linear Programming (ILP) [106], la non-linéarité de notre fonction d'objectif, due aux différents traitements d'interférence pose ici problème. Résoudre un problème NLP est en réalité NP-Hard [107]. Pour cette raison, nous n'avons pas d'autre choix que de considérer des approches sous-optimales, basées sur des algorithmes génétiques

[108, 109, 110]. Nous détaillons en Section 6.4.3 l'algorithme génétique que nous proposons pour résoudre de façon sous-optimale notre problème d'optimisation complet.

2.2 Plan de la thèse

La thèse se décompose en deux parties, chacune d'entre elle incluant deux chapitres

- **Partie 1:** Efficacité énergétique dans les réseaux tolérants à la latence proactifs.

  Chapitre 3: Dans la première moitié de la thèse, nous investiguons un modèle de réseau tolérant à la latence proactif. En particulier, nous cherchons à mettre en évidence les concepts clefs, ainsi que les outils mathématiques nécessaires à la résolution de tels problèmes d'optimisation. Dans le Chapitre 3, nous proposons un exemple illustratif. In Section 3.3, nous définissons le concept de connaissance du futur et démontrons comme le système peut exploiter une telle connaissance du futur pour réaliser une optimisation énergétique. L'analyse mathématique du problème est ici détaillée de manière extensive. Nous passons également en revue plusieurs scenarios de connaissances du futur du plus idéal au plus incomplet, avec également des scénarios où la connaissance du futur est imparfaite, incomplète ou statistique. Ces scénarios sont plus explicitement détaillés dans la Section 3.4. Dans la Section 3.5 nous mettons en évidence, via des simulations numériques, le gain de performance potentiel offert par une approche tolérante à la latence, ainsi que les gains respectifs des divers scénarios de connaissance du futur. Nous démontrons plus particulièrement que le gain dépend de la variation des canaux et qu'il dépend de la capacité du système à discerner les moments pendant lesquels le canal est bon, des moments où le canal est mauvais. Il apparait également que les scénarios de connaissance imparfaits (statistiques et/ou incomplets) sont eux aussi capables d'amener des gains significatifs comparé au scénario état de l'art en l'absence complète de connaissance du futur. Dans certains de ces scénarios, la performance du système approche celle de l'optimal, obtenu quand le système a une connaissance parfaite du futur.

  Chapitre 4: Dans le Chapter 4, nous investigons l'extension à un contexte multi-utilisateurs du problème précédent. L'analyse du problème d'optimisation, détaillée en Section 4.2, révèle que le problème peut être modélisé comme un jeu stochastique non-coopératif, qui devient rapidement difficile à résoudre quand les dimensions du système deviennent trop importantes. Nous détaillons plus explicitement les raisons de cette complexité mathématique et mettons en évidence les méthodes classiques permettant de la contourner malgré tout. En particulier, nous détaillons en Section 4.5, comment nous pouvons exploiter les récentes avancées de la théorie des jeux à champs moyens, pour définir un jeu équivalent à champ moyen, à plus faible complexité mathématique. Une procédure itérative permettant de calculer l'équilibre dans notre jeu à champ moyen est détaillée et des simulations numériques permettent de mettre en évidence les performances des stratégies optimales obtenues via le jeu à champ moyen. Deux stratégies état de l'art sont considérées et détaillées. Pour divers modèles de canaux, des simulations numériques sont proposées. Ces simulations démontrent tout d'abord la validité de l'approximation jeu à champ moyen, en terme d'optimalité dans des scénarios où l'optimum peut être explicitement calculé via une méthode KKT. Dans des scénarios où les canaux varient, l'optimum n'est plus simplement calculable. Les stratégies jeu à champ moyen sont néanmoins nettement plus efficaces que les stratégies de référence. Ces résultats numériques mettent en évidence l'intérêt que le système peut avoir à exploiter la tolérance à la latence, ainsi qu'une éventuelle connaissance du futur, tant il amène à des gains significatifs en termes d'efficacité énergétique.

- **Partie 2:** Classification d'interférence, matching d'interféreurs et Virtual Handover

  Chapitre 5: Nous investiguons dans ce chapitre, comment les récentes avancées dans le domaine de classification d'interférence peuvent être exploitées pour améliorer la performance du système à configuration de puissance constante. Ce faisant nous améliorons l'efficacité énergétique du système. Dans le Chapitre 5, nous présentons tout d'abord un court tutoriel sur les différentes techniques de gestion d'interférence. Nous investiguons ensuite en Section 5.3 le problème de classification d'interférence optimale en contexte de 2-users Gaussian Interference Channels. Dans la Section 5.4, nous choisissons d'exploiter les résultats obtenus dans un nouveau problème d'optimisation, consistant en un matching d'interféreurs, capables de traiter de l'interférence via classification d'interférence. Nous détaillons ce nouveau problème en Section 5.4. Nous expliquons aussi rapidement dans ce chapitre, les raisons pour lesquelles la classification d'interférence en contexte de M-users Gaussian Interference Channels s'avère difficile et reste aujourd'hui une question ouverte. Bien que le problème de matching devienne NP-Hard quand M dépasse 2, nous résolvons dans ce chapitre le problème de matching quand $M > 2$. Des simulations numériques sont finalement fournies pour illustrer les gains potentiels offerts par les techniques de classification d'interférence et de matching d'interféreurs, comparé à des techniques de gestion d'interférence classiques.

Chapitre 6: Dans le Chapitre 6, nous investiguons le concept de 'virtual handover': lorque la classification d'interférence est considérée, nous devons réenvisager la façon dont sont assignés les utilisateurs aux points d'accès, étant donné que le point d'accès fournissant le meilleur SNR n'est plus nécessairement le meilleur. Nous prouvons cela via un exemple illustratif en Section 6.2.2. Il faut alors reprendre l'analyse des problèmes d'optimisation précédent en considérant à présent que le système peut librement réassigner les utilisateurs aux points d'accès. On obtient alors un problème à trois degrés de liberté : traitement de l'interférence, matching d'interféreurs, et assignations d'utilisateurs aux points d'accès. Nous procédons étape par étape. Dans la Section 6.2.3, nous mettons à jour nos résults de classification d'interférence en contexte 2-users Gaussian Interference Channels, lorsque le virtual handover est considéré. La nouvelle classification qui en découle peut alors être réutilisé pour le problème de matching à $M = 2$ points d'accès et $2N$ interféreurs non-assignés, comme détaillé en Section 6.3. Le matching est résolu de façon optimale via l'aglorithme de Edmonds, issu de la théorie des graphes. L'extension du problème à $M > 2$ points d'accès est également considérée. Pour ce faire, nous proposons tout d'abord une classification d'interférence sous-optimale en contexte de M-users Gaussian Interference Channels, que nous détaillons en Section 6.4.2. Il faut alors considéré, comme expliqué en Section 6.4.2, un problème d'optimisation à deux degrés de liberté : matching d'interféreurs et assignation d'utilisateurs aux points d'accès. Ce problème s'avère appartenir à une classe de problème dite de Non-Linear Programming qui s'avère être NP-Hard: nous proposons alors en Section

6.4.3, un algorithme génétique pour résoudre ce problème. Les simulations numériques qui concluent ce chapitre permettent alors de mettre en évidence des gains de performance notables, obtenus grâce à nos diverses techniques de classification d'interférence, de matching d'interféreurs et de virtual handover.

2.3 Publications

Les travaux de cette thèse ont donné lieu aux publications scientifiques suivantes:

Papiers de journaux

- **J1:** [112] De Mari, M. and Calvanese Strinati, E. and Debbah, M., '5G Network Performance Optimization basedon Matching and Interference Classification', *submitted IEEE Transaction on Wireless Communications, 2015.*
- **J1:** [113] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Mean Field Games Power Control for Proactive Delay-Tolerant Networks', *to be submitted to IEEE Transactions on Wireless Communications, 2015.*

Papiers de conférences

- **C1:** [114] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Concurrent data transmissions in green wireless networks: When best send one's packets?', *9th International Symposium on Wireless Communications Systems (ISWCS), 2012.*
- **C2:** [115] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Energy-efficiency and future knowledge tradeoff in small cells prediction-based strategies', *12th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), 2014.*
- **C3:** [116] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Two-regimes interference classifier: An interference-aware resource allocation algorithm', *IEEE Wireless Communications and Networking Conference (WCNC), 2014.*
- **C4:** [117] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Matching Coalitions for Interference Classification in Large Heterogeneous Networks', *IEEE 25th Annual International Symposium on Personnal Indoor and Mobile Radio Communications, 2014.*

- **C5:** [118] De Mari, Z. Becvar, M. and Calvanese Strinati, E. and Debbah, M., 'Interference Empowered 5G networks', *5GU Conference, 2014.*

- **C6:** [119] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'Mean Field Games Power Control for Proactive Delay-Tolerant Networks', *to be submitted, 2015.*

- **C7:** [120] De Mari, M. and Calvanese Strinati, E. and Debbah, M., 'A Game Theoretical Approach to Interference Classification in M-Users Gaussian Interference Channels', *to be submitted, 2015.*

Patents

- **P1:** [121] E. Calvanese Strinati, M. De Mari, M. Debbah, 'PROCEDE DE PRISE DE DECISION D'UN TRANSFERT DE COMMUNICATION DANS UN CONTEXTE INTERFERENTIEL INTRA-BANDE', Filed in February 2015.

Note: Tous les papiers, à l'exception du brevet sont disponibles sur mon site web, à l'adresse suivante `http://matthieu-de-mari.fr/publications-and-papers/`.

Chapter 3

Future Knowledge in Proactive Delay-Tolerant Communications

3.1 Introduction

In this first chapter, we provide a simple illustrative example which introduces two main concepts that are used in the first half of this thesis: future knowledge and proactive resource allocation in delay-tolerant networks. More specifically, we consider a scenario of delay-tolerant transmission with a single user and analyze how the system can exploit a given a priori knowledge about its future context of transmission, in order to adapt its transmission power to the present and future contexts, thus enhancing its energy efficiency. We investigate different kinds of future knowledge ranging from zero a priori knowledge (in this reference scenario, the resource allocation scheme adapts to the present knowledge only, and is thus called reactive) to perfect a priori knowledge (the system knows everything perfectly, long time in advance, which is the optimal configuration). Several partial future knowledge scenarios (statistical, incomplete, erratic, etc. knowledge) are also studied in this chapter.

The conducted analysis of the optimization problem reveals how the system can exploit a given future knowledge in order to minimize its total power consumption, while ensuring a given transmission constraint before a certain deadline. Numerical results provide good insights on the beneficial gains offered to the system by proactive resource allocation, when coupled with different kinds of future knowledge hypotheses.

The remainder of this chapter is organized as follows. After introducing the motivations, contributions and related works to it, we describe in Section 3.2 the delay-tolerant transmission model and its related optimization problem, which is considered through this chapter. In Section 3.3, the theoretical analysis is conducted and it leads to an iterative backward process, which allows to define the optimal transmission strategy to be used at any present time, for any present context and any given future knowledge. In Section 3.4, we describe the different kinds of future knowledge considered in this chapter. In Section 3.5, numerical results provide good insights on how the system benefits from proactive resource allocation and each kind of future knowledge. Finally, we discuss in Section 3.6 the conclusions of the presented work, its limits and introduce the following chapter, which extensively details the extension of the presented analysis to a multiuser scenario, with perfect knowledge only.

3.1.1 Motivations and Related Works

Nowadays, operators struggle to support the massive data traffic growth in a sustainable and economical way [1]. In that sense, operators aim at reducing the network power consumption while maintaining a satisfying quality of service. Among the several trade-offs that have been identified [19, 122], we focus in this chapter on the power efficiency vs. latency trade-off which lead to the delay-tolerant networks and transmission scheduling concepts (refer to [123, 21, 124, 125, 25, 23, 126, 127] for examples). Such approaches make even more sense as a non-negligible part of the current transmissions can be labeled as non-urgent []: for such non-urgent transmissions, it is not necessary to transmit instantaneously, the system might instead be offered a delay, and can decide freely how and when it will transmit, the only constraint being that the transmission must be completed before a given deadline. The system is then free to schedule its transmissions, and can, for instance, aim at completing its required transmission at a minimal cost. Moreover, this delay tolerance allows the system to better handle the network congestion, as suggested in [20, 26, 27].

At the same time, recent advances on learning and data mining have shown that human behavior was highly and accurately predictable [29, 28, 128]. As a consequence, recent works have then looked forward to coupling scheduling techniques with future context predictions, in order to enhance the network performance leading to so-called proactive networks, as introduced in [35, 36, 37]. Significant diversity gains were analytically demonstrated, thus illustrating the significant potential benefit of proactive networks. In these papers [38, 39, 40] for example, the system is able to formulate predictions on the upcoming requests and user mobility: by coupling it with a radio map giving measured reception quality at different locations, the system can then formulate predictions on the expected future transmission contexts [32, 33, 34]. Based on these predictions, it then adapts its present transmission settings, in order to limit its own outage probability.

Most of the time, when facing power control and optimization problems, the problem turns out to be convex. We then may refer to the theory of convex optimization, for which the book of Boyd and Vandenberghe [43] is certainly one of the most cited and complete reference. In case of non-convexity, some papers either investigate how the problem can be assumed convex, or propose specific algorithms to deal with these non-convex scenarios. In the general case, the optimal solution is attained by computing the Lagrangian associated with the optimization problem. The Lagrangian links the objective function to equality and inequality constraints functions by using Lagrange multipliers. Karush-Kuhn-Tucker(KKT) conditions [44] are used to derive the optimal solution to the problem. Kandukuri and Boyd [12] for instance address both the minimization of transmitter power subject to constraints on outage probability and the minimization of outage probability subject to power constraints. Another interesting power allocation problem, which consists of maximizing the ergodic capacity of the broadcast channel subject to minimum rate constraints, is addressed in [129, 130].

In this chapter and as in [41, 42], we investigate a scheduler for delay-tolerant transmissions, but consider several kinds of future knowledge. We analyze how the system can exploit any offered future knowledge, in order to minimize its own total power consumption, while ensuring a complete required transmission before a given deadline. Moreover, we provide numerical simulations that assess the potential performance gains offered by a delay-tolerant approach coupled with several scenarios of future knowledge and compare their performance to those of classical reactive resource allocation schemes, i.e. scenarios where no future knowledge is available.

3.1.2 Contributions

The content of this chapter has been published in one conference paper [115]. The innovation and scientific contributions presented in this chapter are three-fold. First, we provide a general definition of what future knowledge consists of. We also define different knowledge scenarios that can be either:

- perfect, statistical, erratic
- complete or incomplete
- reactive or proactive
- capable of learning from the present and past iteration or not.

The complete list of future knowledge scenarios is extensively detailed in Section 3.4.

Second, a general analysis of the optimization problem is provided, that can suit every possible scenario of future knowledge. In some cases, closed-form expressions of the power strategies can be obtained and the performance can be computed explicitly. For the other cases, an iterative backward process is proposed, which is capable of approaching the optimal transmission strategy to be used at the any present time, given the present transmission context, present state and future knowledge available. Finally, we provide numerical results giving good insights on the performance gains of each scenario of future knowledge, compared to the scenario where no future knowledge is available (i.e. the zero knowledge scenario).

Through this simple illustrative example, we provide answers to the following three fundamental questions related to delay-tolerant networks and future knowledge:

- **How can the system exploit some future knowledge?** A possible way for the system to exploit this future knowledge relies on exploiting the power-efficiency latency trade-off. We model a delay-tolerant transmitter, and consider a power control optimization problem, where the objective is to minimize the global power consumption required for completing a fixed transmission before a given deadline. The transmitter is cognitive and can adapt its transmission power to the present transmission context, in real time. The decision process for the optimal power strategy is then affected by the present state (time remaining before deadline, packet size remaining,etc.) but is also able to take into account some piece of future knowledge about the future transmission context.

- **Does future knowledge offer significant performance gains?** The numerical simulations show that there is a significant gain between i) the zero knowledge scenario, which is the worst scenario of future knowledge, since the system does not know anything about the future transmission context, and thus is lower performance bound; and ii) the perfect knowledge scenario, which is the best scenario of future knowledge, since the system has perfect knowledge of the future at any time, and thus is the higher performance bound. Demonstrating that the gain was significant really mattered: if the performance gap had not been significant enough, then looking for future knowledge, and providing it to the system, so that it can exploit it via scheduling and proactive resource allocation would not have made sense. The performance gain would have been limited, and there would have been really little chance that this performance gain would have surpassed the cost of accessing and exploiting this future knowledge (commonly referred to as the 'cost of learning'). This chapter does not include details about how a piece of future knowledge might be acquired, nor does it define the cost of learning for every single future knowledge scenario. Nevertheless, a few details on this topic are discussed in Section 3.6.

- **What kind of future knowledge is really useful to the system?** The conducted analysis shows that the system may greatly benefit from partial future knowledge, and may almost reach the performance of the perfect knowledge scenario. More specifically, it turns out that a good statistical knowledge of the future context can offer significant performance gains. Also, it appears that a short-term knowledge (i.e. precise knowledge about the close future exclusively) can also provide significant performance gains.

This chapter presents a single user analysis of a proactive resource allocation scheme, for different degrees of future knowledge, showing key concepts about proactive resource allocation. It also provides insights about the significance of the potential performance gains offered by the proactive concept, applied to a power efficiency versus latency tradeoff. The extension of the presented model to a multiple competing users model, which leads to a lot more sophisticated analysis, is extensively treated in Chapter 4.

3.2 System Model and Optimization Problem

3.2.1 System Model

In this paper, we consider a simple downlink transmission model (similar to [41, 42]), consisting of one Access Point (AP) and one User Equipment (UE). The AP is required to transmit a data packet to its associated user, within a limited number of Time Slots (TS) T. In the following, we denote the index of any time slot by $t \in \mathcal{T} = \{1, 2, ...T\}$. In this context and as depicted in Figure 3.1, we consider for each time slot, uncorrelated block fading channels, in power units, $h_r(t), t \in \mathcal{T}$, with values in $\mathcal{H}$. We consider that a minimum link quality is always guaranteed, denoted $\epsilon > 0$, i.e. that $\mathcal{H} =]\epsilon, \infty[^T$. This assumption will later appear necessary, when computing strategies for which we do not have future knowledge. We assume, that the channel realizations are random i.i.d processes, distributed according to Probability Density Functions (PDF) $D^1_{real}(h), ...D^T_{real}(h)$, with $h \in \mathcal{H}$. We assume, that the present channel can be perfectly estimated, at the transmitter side, at the beginning of each time slot, and that it remains static for the complete duration of the time slot Δt. We also denote by $Q(t)$, the number of remaining bits at the end of time slot t. The initial amount of data, denoted $Q(0) > 0$, is known at the beginning of the first time slot. For simplicity, we assume that no other requests are allowed to enter the system until the end.

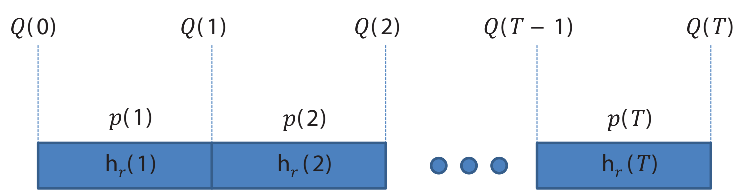

Figure 3.1: System Model: Transmission scheme

We assume that the AP can adapt its power of transmission $p = (p(t))_{t \in \mathcal{T}}$, at will. We denote $p(t) > 0$, the transmission power used during time slot t, which is defined by the transmitter at the beginning of each time slot t. We assume that the transmission rate matches the channel capacity, i.e. that the remaining packet size at the end of time slot t, $Q(t)$ decreases according to equation (3.1):

$$\forall t \in \mathcal{T},\, Q(t) = Q(t-1) - B\log_2\Big(1 + h_r(t)p(t)\Big)\Delta t \tag{3.1}$$

Where Δt and B are constants denoting respectively the duration of the time slot and the bandwidth of the channel.

In addition to the perfectly estimated present channel, we assume that the system is given a certain future knowledge, at each present time t, about the future channel realizations. More specifically, we denote $\forall t, i \in \mathcal{T}, i > t$, $D_i^t(h), h \in \mathcal{H}$ the prediction made by the system, at the beginning of TS t about the channel realization $h_r(i)$ that will occur on TS i. Mathematically speaking, the predictor s$D_i^t(h)$ is a PDF of the channel realization $h_r(i)$, given to the system at TS t. The system then makes a 'guess' about the future channel realization $h_r(i)$, and 'guesses' that the future channel realization $h_r(i)$ follows the PDF $D_i^t(h)$. A perfect knowledge at time t of the future realization which is supposed to happen on TS $i > t$ would then correspond to $D_i^t(h) = \delta_{h=h_r(i)}$, with $\delta_{h=h_r(i)}$ being the Dirac distribution centered at $h_r(i)$.

3.2.2 Optimization Problem Formulation

Within this context, we consider the following constrained optimization problem: the AP has to define a transmission power strategy p that allows a complete transmission of a data packet, whose initial size $Q(0) > 0$ is known at $t = 0$, before deadline T, at a minimal cumulated power cost. According to our model, achieving a complete transmission at deadline T is then expressed by $Q(T) = 0$. The problem can be formally rewritten as the following optimization problem (3.2):

$$p^* = (p^*(1), p^*(2), ...p^*(T)) = \arg\min_p \left[\sum_{k=1}^{T} p(k)\right]$$

$$\text{s.t., } Q(T) = Q(0) - \sum_{k=1}^{T} B\log_2\Big(1 + h_r(k)p(k)\Big)\Delta t = 0 \tag{3.2}$$

And $\forall t, i \in \mathcal{T}, i > t$, $D_i^t(h)$ are the predictors given to the system about $h_r(i)$, at the beginning of TS t.

In Section 3.3, we focus on how the system exploits its given information about the future $D_i^t(h)$, with $i > t$, to compute at the beginning of each TS

t, the optimal power to be used, $p^*(t)$. The optimal power to be used at the beginning of each time slot, obviously depends on the packet size remaining at the beginning of the present time slot $Q(t-1)$, but also on both the present channel realization and the prediction that we consider for the channel realizations on the remaining time slots. We detail the future knowledge scenarios and predictions associated to these scenarios in Section 3.4.

3.3 Analysis of the Optimization Problem (3.2)

At the beginning of each TS t, the system is revealed the present channel realization for the TS t, denoted $h_r(t)$. Based on this present information and for a given knowledge of the future, represented by the predictors $\mathcal{D}_t = (D^t_{t+1}(h), ...D^t_T(h))$, the system has to define the optimal power $p^*(t)$ to be used for TS t, assuming the remaining packet to transmit until deadline T is of $Q(t-1)$. For any present channel realization $h_r(t)$, remaining packet size $Q(t-1)$ and predictors $\mathcal{D}_t$, the optimal power $P^*(t \mid h_r(t), Q(t-1))$ to be used at time t is defined at the beginning of TS t, as the power that will minimize the expected total power consumption, i.e.:

$$P^*(t \mid h_r(t), Q(t-1)) = \arg\min_p \left(p + \mathbb{E}\left[\sum_{k=t+1}^{k=T} p(k) \mid Q(t) \right] \right) \tag{3.3}$$

With $Q(t) = Q(t-1) - B\log_2\Big(1 + h_r(t)p\Big)$.

The difficulty of the backward dynamic optimization process [45] relies in being able to estimate the cost-to-go function $S(t+1 \mid Q(t)) = \mathbb{E}\left[\sum_{k=t+1}^{k=T} p(k) \mid Q(t)\right]$ at time t, since it depends on all the random channel realizations of the $T-t$ remaining time slots, $(h(t+1), ..., h(T))$. This cost-to-go function represents the expected additional cost that the system should pay if the remaining packet size at the end of TS t is $Q(t)$, assuming the future channel realizations will follow the predictors $\mathcal{D}_t$. However, we can notice that the cost-to-go function is convex in p. We can then compute the optimal power strategy to be used in any configuration, sequentially, with dynamic backward programming, as in [43]. The optimal power to be used on TS T, $P^*(T \mid h, Q)$ if the channel realization is $h_r(T) = h$ and the remaining packet $Q(T-1) = Q$ can be easily computed, as the power necessary to complete the transmission on the last time slot T, i.e.

$$P^*(T \mid h, Q) = (2^{(\frac{Q}{B\Delta_t})} - 1)\frac{1}{h} \tag{3.4}$$

At $t = T-1$, the system can compute $S(T \mid Q) = \mathbb{E}[P(T \mid Q)]$, as the expected power cost on the last time slot:

$$S(T \mid Q) = \int_{h\in\mathcal{H}} D_T^t(h)P^*(T \mid h, Q)dh \tag{3.5}$$

Which leads to:

$$S(T \mid Q) = (2^{(\frac{Q}{B\Delta_t})} - 1)\mathbb{E}\left[\frac{1}{h(T)}\right] \tag{3.6}$$

Note that $\mathbb{E}\left[\frac{1}{h_r(T)}\right]$ is not finite if the channels have Rayleigh fading (i.e. $h_r(T)$ is exponentially distributed), unless the set for possible channel realizations $\mathcal{H}$ is truncated as described in [46]. In fact, this is the main reason why we have considered a minimal channel realization $\epsilon > 0$ and considered the set of admissible channel realizations $\mathcal{H}$ to be $]\epsilon, \infty[^T$.

Based on the previous results, we can solve the optimization problem (3.3), via the following iterative process. For k from $T-1$ to t, we can sequentially compute:

$$P^*(k \mid h, Q) = \arg\min_p \left[p + S(k+1 \mid Q')\right] \tag{3.7}$$

With $Q' = Q - B\log_2(1+ph)$. This is a one-dimensional convex optimization problem, in p, which is easy to solve, using dichotomic search for example [131], when a closed-form solution can not be easily computed. Based on this, we can compute $S(k \mid Q)$, as:

$$S(k \mid Q) = \int_{h\in\mathcal{H}} D_k^t(h)P^*(k \mid h, Q)dh + \int_{h\in\mathcal{H}} D_k^t(h)S(k+1 \mid Q')dh \tag{3.8}$$

Where $Q' = Q - B\log_2(1+P^*(k \mid h, Q)h)$.

The iterative process is then repeated until $P^*(t \mid h, Q)$ is defined, with t being the present time. For a given present realization of the channel $h_r(t)$ and assuming the remaining packet size is $Q(t-1)$, we can then define the optimal power to be used on time slot t, $p^*(t)$, as:

$$p^*(t) = P^*(t \mid h_r(t), Q(t-1)) \tag{3.9}$$

Using this process, at the beginning of each time slot t, we can compute the optimal power $p^*(t)$, based on the remaining packet size $Q(t-1)$, the present and revealed channel realization $h_r(t)$ and take into account the future channel predictions $(D_i^t(h))_{i>t}$ available at time t. We now focus on the different scenarios of future knowledge and look forward to define, when it is possible, the closed-form expressions of the optimal transmission policies and their global performance.

3.4 Future Knowledge Scenarios

In this section, we introduce the different scenarios of future knowledge and define the predictors $\mathcal{D}_t = (D_i^t(h))_{i>t}$ related to each scenario.

3.4.1 Perfect A Priori Knowledge

In this section, we assume that the system has perfect a priori knowledge of the future channel realizations $h_r(i), i > t$ at $t = 0$ and look forward to defining the optimal power strategy p^*_{om}, solving (3.2) in this scenario. Assuming the system has a perfect knowledge of the future channel realizations is strictly equivalent to the predictions PDFs $D_i^t(h)$ being defined as:

$$\forall i, t \in \{1, ...T\}, i > t, D_i^t(h) = \delta_{h=h_r(i)} \tag{3.10}$$

Where $\delta_{h=h_r(i)}$ is a Dirac distribution centered at $h_r(i)$.

It turns out that the iterative process we defined in Section 3.3, for which we have defined $D_i^t(h)$ as in equation (3.10), leads to the exact same power strategy p^*_{om}, defined by Time-Water-Filling, in Proposition 3.1. Water-filling based power allocation techniques have been widely presented [43, 46, 47] and investigated in the literature [48, 49, 50].

Proposition 3.1. *The power strategy p^*_{om} consists of a time water-filling:*

$$\forall t \in 1, ..., T,\ p^*_{om}(t) = \Big(\mu - \frac{1}{h(t)}\Big)^+ = \max(0, \mu - \frac{1}{h_r(t)}) \tag{3.11}$$

Where μ is the unique water-level, verifying:

$$Q(0) = \sum_{t=1}^{T} B \log\left(1 + h_r(t) p^*_{om}(t)\right) \Delta t \tag{3.12}$$

Proof. The complete proof can be found in Appendix 8.1 □

Since there is no better future knowledge than a perfect one, the strategy p^*_{om} gives the optimal performance bound, that can be achieved for any realization of the channel $h_r = (h_r(1), ..., h_r(T))$ and any constrained data transmission, with initial size $Q(0) > 0$ and deadline T. The closed-form expression of the power strategies is not easy to define, because it depends on the number of time slots active for transmission $\mathcal{N}(Q(0), h)$, defined as:

$$\mathcal{N}(Q(0), h) = card\Big\{p_{om}(t) > 0 \mid t \in \{1, ...T\}, Q(0), h\Big\} \tag{3.13}$$

Assuming, the channel realizations follow the PDFs $D^1_{real}(h), ...D^T_{real}(h)$, computing the closed form-expression of $\mathbb{E}\left[\sum_{k=1}^{k=T} p(k) \mid Q(0)\right]$ in such a scenario is also complicated, since it depends on $\mathcal{N}(Q(0), h)$ as well. For this reason, the unique water-level μ is usually computed using a dichotomic search on equation (3.12). An illustration of the water-filling principle is given on Figure 3.2. For every TS t, if the channel realization $h_r(t)$ is poor compared to the water-level, then it can not be filled by water and the 'hole' remains empty: it follows immediately that $p^*_{om}(t) = 0$. Whereas all other 'holes' are filled by a level of water corresponding to the difference between $\frac{1}{h_r(t)}$ and thus the baseline water level μ, and $p^*_{om}(t) = \mu - \frac{1}{h_r(t)} > 0$.

3.4.2 Zero Knowledge: Worst-Case Scenario

We define by zero knowledge strategy, the power strategy p^*_{zk} that would be implemented, if the system is given the least possible information about its future context and is unable to perform any predictions on the future channel realizations, i.e. it only knows that $\forall t, h_r(t) \in \mathcal{H}$. In this scenario, the next future channel realizations remain unknown to the AP, until the beginning of each time slot, where each present channel is revealed. When facing an unknown future, the safest strategy consists of defining the power $p^*_{zk}(t)$ to be used at the beginning of each time slot t, for a revealed channel realization $h_r(t)$, by assuming that the future realizations will lead to the worst possible configuration. This power allocation scheme is then completely reactive, in the sense that it is not able to exploit any information about the future, and reacts only to the channel information revealed at the present time.

In this context, the best power strategy $p^*_{zk}(t)$ to be implemented on time slot t corresponds to the first element of the optimal power strategy $p = (p(t), ..., p(T))$, solving the following min-max optimization problem.

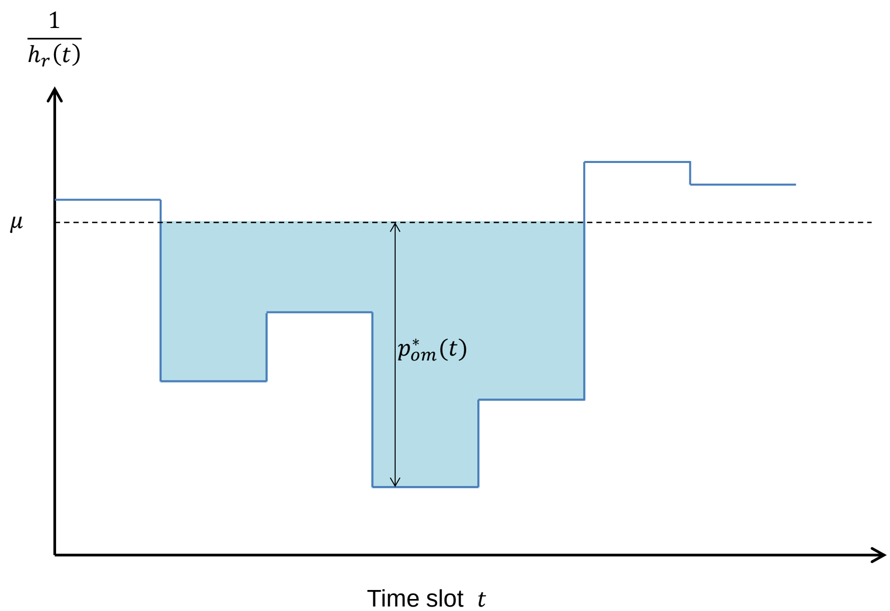

Figure 3.2: Illustration of the water-filling concept: computing the water level μ and the power strategy p^*_{om}

$$\begin{aligned} &\min_{p} \max_{\substack{(h_r(t+1),\ldots h_r(T)) \\ \in \mathbb{H}^{(T-t)}}} \left[\sum_{k=t}^{k=T} p(k)\right] \\ &\text{s.t., } Q(t-1) = \sum_{k=t}^{T} B \log\left(1+h_r(k)p(k)\right)\Delta t \end{aligned} \tag{3.14}$$

The worst possible future configuration can be easily defined as the case where all the unknown future channel realizations $(h_r(t+1), \ldots h_r(T))$ take the smallest value ϵ. In this case, we have $\forall i, t \in \mathcal{T}, i > t, D_i^t(h) = \delta_{h=\epsilon}$ and the optimization problem (3.14) can be rewritten as:

$$\min_{p=(p(k))_{k \geq t}} \left[\sum_{k=t}^{k=T} p(k)\right]$$

$$\text{s.t., } Q(t-1) = \sum_{k=t}^{T} B \log\left(1+h_r(k)p(k)\right)\Delta t$$

$$\text{With } h_r(t) \in \mathcal{H} \text{ known, and } \forall k > t,\, h_r(k) = \epsilon \tag{3.15}$$

The optimization problem leads to the following time water-filling solution:

$$\forall k \geq t,\, p(k) = \begin{cases} \max(0, \mu - \frac{1}{h_r(t)}) & \text{if } k = t \\ \max(0, \mu - \frac{1}{\epsilon}) & \text{else.} \end{cases} \tag{3.16}$$

Where μ is the unique water-level satisfying:

$$Q(t-1) = \sum_{k=t}^{T} B \log\left(1+h_r(k)p(k)\right)\Delta t \tag{3.17}$$

Proposition 3.2. *The power strategy p^*_{zk} is then defined as:*

$$p^*_{zk}(t) = \left(2^{\frac{(Q(t-1)-Q(t))}{B\Delta t}} - 1\right)\frac{1}{h_r(t)} \tag{3.18}$$

Where the packet remaining at time t, $Q(t)$ decreases with t and is given by:

$$\begin{aligned} Q(t) &= \left(\tfrac{T-t}{T-t+1}\left(Q(t-1) - B\log_2\left(\tfrac{h_r(t)}{\epsilon}\right)\right)\Delta t\right)^+ \\ &= \left(\tfrac{T-t}{T}Q(0) - \textstyle\sum_{i=1}^{t} \tfrac{T-t}{T-i+1} B\log_2\left(\tfrac{h_r(i)}{\epsilon}\right)\Delta t\right)^+ \end{aligned} \tag{3.19}$$

Proof. For elements of proof, refer to Appendix 8.2 □

Within this framework, we show, that an AP can smartly exploit the limited knowledge, consisting of each channel realization revealed at the beginning of each time slot, and adapt its power strategy in a reactive way. However, the system pessimistically assumes the worst scenario for the future realizations of the channel. Because of this, the strategy p^*_{zk} appears unable to fully exploit the latency vs. power efficiency tradeoff and, as a consequence, the global performance, in terms of energy-efficiency, of the zero knowledge strategy p^*_{zk} is poor compared to the omniscient one p_{om}. However and for the same reasons we pointed out in the previous section, computing the closed-form expression of the expected power consumption of this strategy leads to serious complications.

3.4.3 Equal-bit Strategy

Another power strategy one could implement, in a scenario where no knowledge of the future is available, is the equal-bit strategy. Basically, the scheduler transmits $\frac{Q(0)}{T}$ bits during each time slot, no matter what the present or future channel realizations might be.

Proposition 3.3. *The power strategy p^*_{eb} is then defined as:*

$$p^*_{eb}(t) = \left(2^{\frac{(Q(0))}{TB\Delta t}} - 1\right) \frac{1}{h_r(t)} \tag{3.20}$$

This scheduler has an expected total power consumption $\mathbb{E}\left[\sum_{k=1}^{k=T} p^*_{eb}(k) \mid Q(0)\right]$, which can be immediately defined as:

$$\mathbb{E}\left[\sum_{k=1}^{k=T} p^*_{eb}(k) \mid Q(0)\right] = (2^{\frac{Q(0)}{TB\Delta t}} - 1) \sum_{i=1}^{T} \int_{h\in\mathcal{H}} D^i_{real}(h)dh \tag{3.21}$$

This strategy is able to take into account the number of extra time slots available for transmission, but does not consider any future channel predictions. As a consequence, the system is unable to smartly distribute the power over the time slots with good channel realizations, while avoiding transmissions on the time slots with poor channel realizations, as a time water-filling would. In the end, its global energy efficiency performance is expected to be poor compared to a scenario where such a future knowledge, even partial, is provided to the system.

3.4.4 Statistical Knowledge about the Future Channel Realizations

In this section, we assume that the system can access a statistical knowledge about the future channel realizations. In this scenario, we assume that the system is not able to tell exactly, what the future channel realizations will be. Instead, it can access the exact channel statistics $(D^i_{real}(h))_{i\in\{1,\dots,T\}}$, that the channel realizations followed, which means that:

$$\forall i, t \in \mathcal{T}, i > t, D^t_i(h) = D^i_{real}(h) \tag{3.22}$$

The iterative process we defined in Section 3.3, uses the available predictions on channel realizations $D^t_i(h)$, $\forall i, t \in \mathcal{T}, i > t$. This strategy allows to take into account the possible channel realizations for the upcoming time slots. This way, the present power strategy p^*_{st}, whose elements are computed at the beginning of each time slot, is able to take into account both the number of remaining time slots $T-t$ and some piece of statistical information about what the remaining future channel realizations might be.

Note that we have considered a scenario where the system would be able to adjust and refine its statistical estimations, at the beginning of each time slot. In fact, we could model such a scenario, by considering that our predictions $D_i^t(h)$ get more and more accurate, when we get closer to the future channel realization, i.e. when the present time slot t gets closer to the time slot i. In that case, it is possible to model a predictor whose accuracy increases, i.e. $D_i^t(h)$ converges to the best predictor one could make about the channel realizations on time slot i, $\delta_{h=h_r(i)}$, when t becomes closer and closer to i. We do not extensively investigate such a class of statistical strategies in this chapter, but we discuss about them in Section 3.6.

3.4.5 Short-term Knowledge

An other scenario of future knowledge that the system can access consists of a short-term knowledge. In this scenario, we assume that the system is able to estimate accurately the present channel realization, at the beginning of each time slot, as well as a few upcoming channel realizations. In fact, there are a lot of channel models, that are able to predict the channel realizations for upcoming time slots, by taking into account the present channel realizations, by modeling the channel evolution with Markov chains or auto-regressive models for example [132]. Such predictors might also exploit elements of context (mobility of the user can be tracked by GPS, and is constrained by roads and streets, etc.).

In this section, we assume that the system is able to predict exactly the channel realizations for the upcoming $K \in \mathcal{T}$ time slots, at the beginning of each time slot. This means, that the prediction provided to the system, at the beginning of time slot t, is:

$$\forall i \in \{t+1, ... \min(T, t+K)\}, D_i^t(h) = \delta_{h=h_r(i)} \tag{3.23}$$

In this section, we assume that the system is not provided any information about any eventual future TS $i \in \{t+K+1, ..., T\}$. In such a scenario, we assume that the remaining predictors are defined as in Section 3.4.2: the system will then assume the worst possible channel realization for these remaining time slots, i.e.:

$$\forall i \in \mathcal{T}, i > (t{+}K), D_i^t(h) = \delta_{h=\epsilon} \tag{3.24}$$

In this scenario, our iterative method allows to compute the optimal power $p^*_{sh(K)}(t)$ to be used at the beginning of each time slot t.

3.4.6 Short-term Knowledge coupled with Statistical Knowledge

We can also model the same kind of short-term future knowledge as in Section 3.4.5. But, instead of assuming that the system does not have any information about the remaining time slots and thus assumes the worst possible channel realizations for these, we now assume that the system is given exact statistical knowledge for the remaining time slots, as in Section 3.4.4. This means that the predictors are now defined as:

$$\forall i \in \{t{+}1, ... \min(T, t{+}K)\}, D_i^t(h) = \delta_{h=h_r(i)} \tag{3.25}$$

And

$$\forall i \in \mathcal{T}, i > (t{+}K), D_i^t(h) = D^i_{real}(h) \tag{3.26}$$

The same way, our iterative method allows to compute the optimal power $p^*_{sh(K)+st}(t)$ to be used at the beginning of each time slot t.

3.5 Numerical Results and Performance Insights

3.5.1 Simulation Parameters

In order to numerically evaluate the expectation of the total consumed power for every possible scenario of future knowledge described in Section 3.4, we run Monte-Carlo simulations over $N_{MC} = 1000$ iterations for arbitrary varying values of $\frac{Q(0)}{B\Delta t}$ (which represents the average amount of data to be transmitted on the T time slots) and T (which represents the latency offered to the system). The performances, in terms of total power consumption, of every strategy are numerically estimated and compared, in a scenario of block-fading truncated Rayleigh channels with parameter $\lambda = 1$, and in presence of noise only. Note that the presented simulations parameters can be easily adapted in order to include a given a priori interference. The channel realizations h_r are then defined as follows:

$$\forall t, h_r(t) = \frac{g(t)}{\sigma_n^2} \in \mathcal{H} \tag{3.27}$$

With $\forall t, g(t)$, defined as random iid realizations of block-fading Rayleigh which have been truncated so that $\forall t, h_r(t) \in \mathcal{H}$, as suggested in [46] and σ_n^2 is the noise variance. The complete list of simulation parameters is given in Table 3.1, below.

Parameter	Parameter Value
$\frac{Q(0)}{B\Delta t}$	ranging from 1 to 200
Number of time slots T	25, as in [41, 42]
Number of MC iterations M_c	1000
Channels $g(t)$	Block fading with i.i.d. trunc. Rayleigh(1)
Channel set $\mathcal{H}$	$]\epsilon, \infty[$
ϵ	0.1
Noise variance σ_n^2	-128 dBm, as in [133]

Table 3.1: Simulation parameters

3.5.2 Insights about the Significance of the Potential Performance Gain

Our first concern when considering schedulers that are able to take into account predictions about the future transmission context is to determine how significant the potential energy performance gains might be, between the scenario when a perfect future knowledge is available and a scenario where no future knowledge is given to the system. We provide in Figure 3.3, the packet size evolutions used for one channel realization $h_r = (h_r(1), ..., h_r(T))$, $T = 25$ time slots and $\frac{Q(0)}{B\Delta t} = 200$, in 3 scenarios of interest, namely:

- Perfect knowledge as in Section 3.4.1,
- Zero knowledge as in Section 3.4.2,
- Equal-bit scheduling as in Section 3.4.3,
- and Statistical knowledge as in Section 3.4.4.

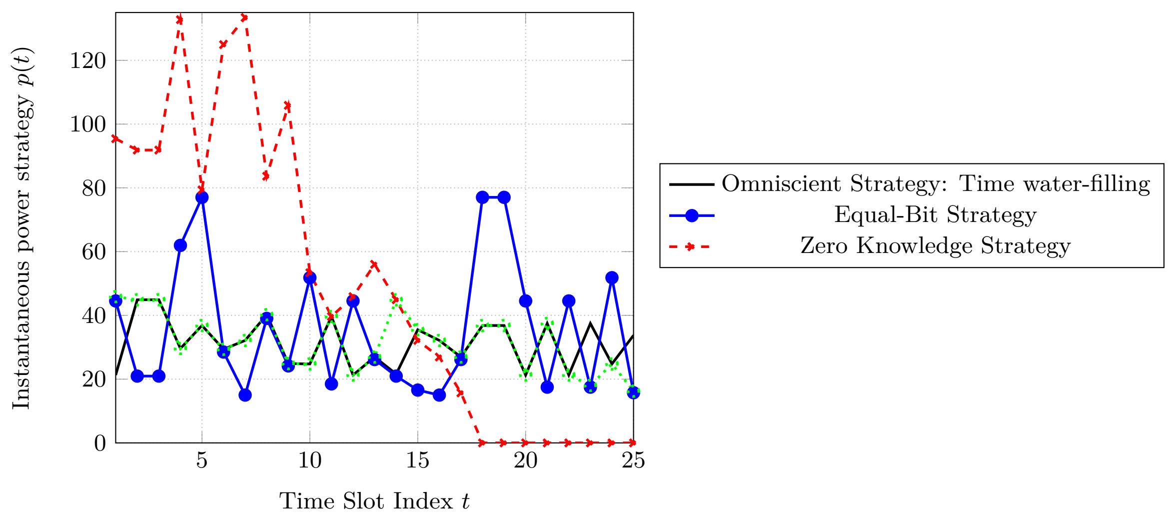

Figure 3.3: Instantaneous power strategies $p(t)$ vs. Time Slot Index t, for several scenarios of future knowledge - $\frac{Q(0)}{B\Delta t} = 100$ and $T = 25$.

—— Fig 2

We also represent in Figure 3.4, the $\alpha_r(sc)$ performance criterion for the zero knowledge ($sc = zk$), equal-bit ($sc = eb$), and statistical knowledge ($sc = st$) strategies. For a given scheduler sc, the $\alpha_r(sc)$ criterion is defined as the ratio between the expectation of the total power consumption for the given scenario sc and the expectation of the total power consumption for the perfect knowledge scheduler. The expectations are calculated empirically, using the $M_c = 1000$ independent Monte-Carlo iterations. The $\alpha_r(sc)$ criterion is then defined as:

$$\alpha_r(sc) = \frac{\mathbb{E}\left[\sum_{t=1}^{T} p^*_{sc}(t)\right]}{\mathbb{E}\left[\sum_{t=1}^{T} p^*_{om}(t)\right]} = \frac{\sum_{n=1}^{M_c}\sum_{t=1}^{T} p^{*(n)}_{sc}(t)}{\sum_{n=1}^{M_c}\sum_{t=1}^{T} p^{*(n)}_{om}(t)} \tag{3.28}$$

Where $p^{*(n)}_{sc}$ is the power strategy related to scheduler sc, used on time slot t, during the Monte-Carlo iteration $n \in \{1, ..., M_c\}$.

As expected, it appears that the zero knowledge strategy is unable to exploit the offered latency offered: since it is assuming the worst possible realizations for the future channels, the power strategy p^*_{zk} transmits using high power in the first time slots, in order to prevent itself from transmitting in what the system expects to be a poor channels future. The resulting scheduler then

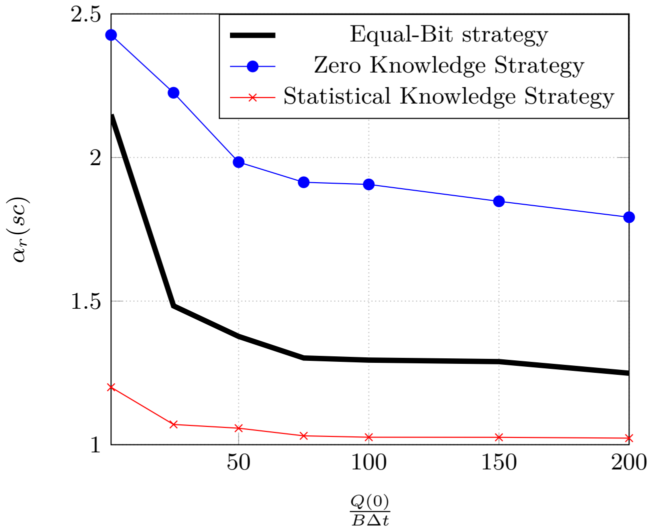

Figure 3.4: Energy Performance vs. $\frac{Q(0)}{B\Delta t}$ - $\frac{Q(0)}{B\Delta t}$ ranging from 1 to 100 and $T = 25$.

rushes the transmission of packets, completing it notably before the deadline. The resulting power efficiency is then poor, since it transmits using only a mall fraction of the T time slots. Although provided with no future knowledge as well, the equal-bit scheduler appears able to exploit the latency offered, but is unable to identify and favor the good channels time slots over the bad ones. Its power efficiency is slightly better than the zero-knowledge one, but still remains poor. The statistical knowledge however can be used by the system to identify if a present channel realization is good compared to the expected possible channel realizations, that could possibly happen in the future. Such a knowledge allows the system to exploit the offered latency, by distributing the powers over all the available time slots, slightly favoring the good channel realizations for transmission. The optimal power strategy is given by the perfect knowledge scenario. We observe that a statistical knowledge allows the system to remarkably approach the optimal power strategy p^*_{om}, thus resulting in a good power efficiency for the power strategy p^*_{st}.

Two notable conclusions can be highlighted from those numerical simulations.

- First, the performance gap between the two schedulers, which were not provided any information about the future channel realizations and the perfect knowledge one is rather significant. This result highlights and assesses the interest of accessing and exploiting a future knowledge in a delay-tolerant transmission systems. Based on Figure 3.4, the ratio of total power consumption of the zero knowledge/equal-bit schedulers to the total power consumption of the perfect knowledge scenario consists of a factor between 1.3 and 2.4, which is significant.

- Second, it turns out that accessing a statistical knowledge allows to approach remarkably the performance bound, obtained in the perfect knowledge scenario. Since accessing a perfect knowledge is unrealistic, it is interesting to notice, that a good statistical knowledge of the future channel realizations (as in [36]), may lead to remarkable performance, closely approaching the optimal one.

3.5.3 Performance Analysis of the Partial Knowledge Scenarios

In this section, we investigate the performance of the two short-term knowledge strategies, described in Sections 3.4.5 and 3.4.6. We compare their performance to the perfect knowledge, zero-knowledge, equal-bit and statistical knowledge schedulers, investigated ion the previous section. To do so, let us first define the following two performance criterion $\alpha_z(K)$ and $\alpha_s(K)$, for any $k \in \{1, ..., T\}$, as follows:

$$\alpha_z(K) = \frac{\mathbb{E}\left[\sum_{t=1}^{T} p^*_{zk}(t)\right] - \mathbb{E}\left[\sum_{t=1}^{T} p^*_{sh(K)}(t)\right]}{\mathbb{E}\left[\sum_{t=1}^{T} p^*_{zk}(t)\right] - \mathbb{E}\left[\sum_{t=1}^{T} p^*_{om}(t)\right]} \tag{3.29}$$

$$\alpha_s(K) = \frac{\mathbb{E}\left[\sum_{t=1}^{T} p^*_{st}(t)\right] - \mathbb{E}\left[\sum_{t=1}^{T} p^*_{sh(K)+st}(t)\right]}{\mathbb{E}\left[\sum_{t=1}^{T} p^*_{st}(t)\right] - \mathbb{E}\left[\sum_{t=1}^{T} p^*_{om}(t)\right]} \tag{3.30}$$

In practice, $\alpha_z(K) \in [0, 1]$ tells the amount of the performance gap gained by the short-term knowledge scheduler. The performance gap here corresponds to the performance difference between the zero knowledge and perfect knowledge scenario. If $\alpha_z = 1$, then its performance matches the performance of the perfect knowledge scheduler. If $\alpha_z = 0$, then it matches the performance of the zero knowledge scheduler. Same goes for α_s, except that it compares the short-term knowledge coupled with statistical knowledge scheduler with the perfect knowledge and statistical knowledge schedulers. Also, when $\alpha_s = 0$, its performance matches the performance of the statistical knowledge instead of the zero knowledge one.

We have represented on Figure 3.5 the evolution of the α_z criterion for values of K ranging from 1 to T (i.e. $\frac{K}{T}$ ranging from $\frac{1}{T}$ to 1), $M_c = 1000$, $T = 25$ and $\frac{Q(0)}{B\Delta t} = 100$. We also represented the performance of the equal-bit scheduler α_{eb}, whose value is constant with respect to K, and is defined as follows:

$$\alpha_z(K) = \frac{\mathbb{E}\left[\sum_{t=1}^{T} p_{zk}^*(t)\right] - \mathbb{E}\left[\sum_{t=1}^{T} p_{eb}^*(t)\right]}{\mathbb{E}\left[\sum_{t=1}^{T} p_{zk}^*(t)\right] - \mathbb{E}\left[\sum_{t=1}^{T} p_{om}^*(t)\right]} \tag{3.31}$$

In Figure 3.6, we have represented the evolution of the α_s criterion for the same simulation parameters.

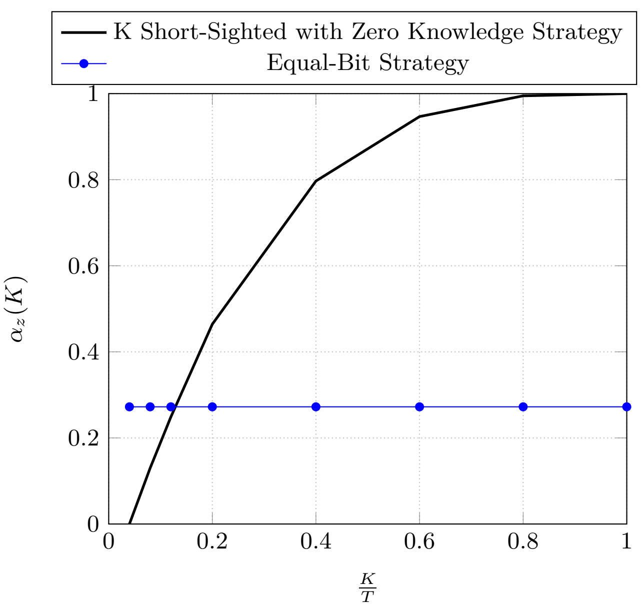

Figure 3.5: $\alpha_z(K)$ criterion vs. $\frac{K}{T}$, $\frac{Q(0)}{B\Delta t} = 100$ and $T = 25$.

As expected, we observe on Figure 3.5 that when $K = 1$, the scheduler knowledge matches the zero knowledge one, and thus has the same performance ($\alpha_z = 0$). also, when $K = T$, the scheduler has a perfect a priori knowledge of the future at $t = 0$, its performance matches the perfect knowledge one. We also observe that α_z increases with K and the increase is not linear: on the contrary, we observe that the larger K becomes, the less significant the extra performance

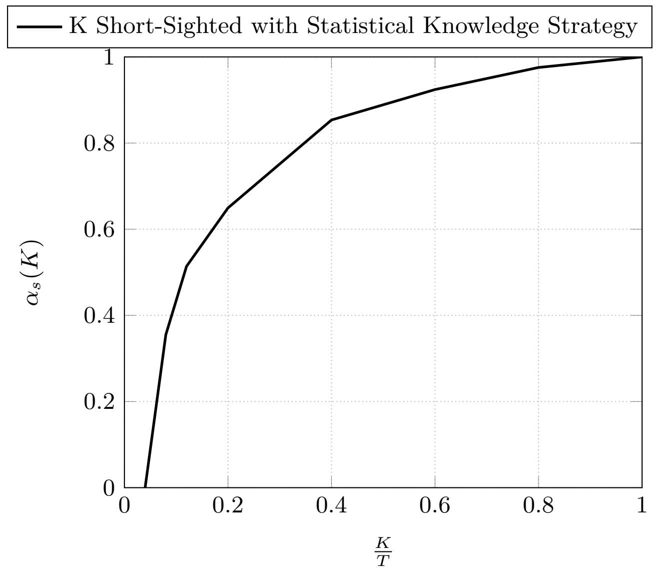

Figure 3.6: $\alpha_s(K)$ criterion vs. $\frac{K}{T}$, $\frac{Q(0)}{B\Delta t} = 100$ and $T = 25$.

gain is. As a consequence, it appears that providing an even really short-term knowledge ($K > 1, K << T$) might actually lead to significant performance gains. The same observations can be made, on Figure 3.6, with the α_s criterion and the scenario of short-term knowledge coupled with statistical knowledge.

Finally, we discuss the performance of several future knowledge schedulers The following schedulers have been considered:

- Zero Knowledge Scheduler
- Equal-Bit Scheduler
- Statistical Knowledge Scheduler
- K Short-Sighted, with Zero Knowledge Scheduler ($K = 2, 3, 5$)
- K Short-Sighted, with Statistical Knowledge Scheduler ($K = 2, 3$)

Figure 3.7 shows the ratio between the average energy performance of each strategy and the average performance of the omniscient scheduler for several values of $\frac{Q(0)}{B\Delta t}$ and $T = 25$.

It appears that the performance of the short-term knowledge scheduler is better than the one of the equal-bit scheduler, even for small values of K (in our case, when $\frac{K}{T} \geq 0.15$), and we observe that the performance of the K short-term scheduler improves as long as K increases, with decreasing gain every time. The performance of the statistical Knowledge scheduler is again remarkably close to the optimal one, as demonstrated also in Section 3.5.2. Adding a short-term knowledge to the statistical knowledge contributes to improve even more the performance of the scheduler, which rapidly tends to the optimal performance achieved by the perfect knowledge scheduler.

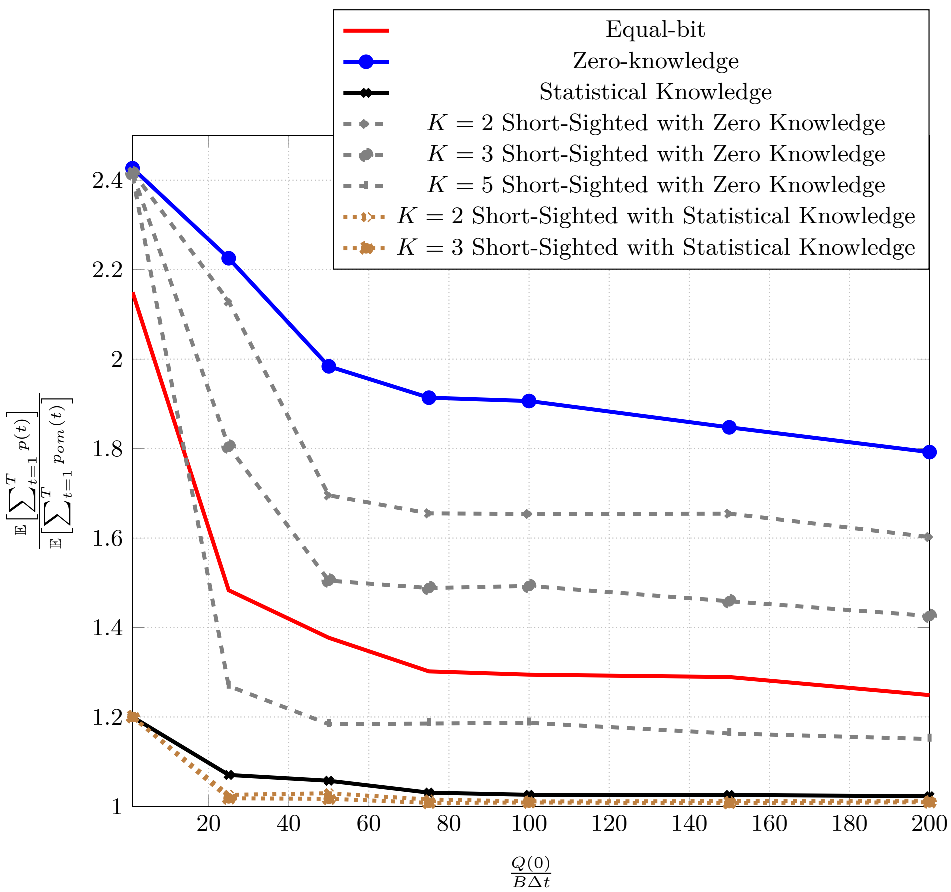

Figure 3.7: Energy Performance vs. $\frac{Q(0)}{B\Delta t}$ - $\frac{Q(0)}{B\Delta t}$ ranging from 1 to 200 and $T = 25$.

3.5.4 Insights About the Performance Gap Evolution wrt the Channel Variations

In this section, we investigate an interesting property of the performance gap observed in Section 3.5.2. This performance gap seems to vary with the channel variations. More specifically, the system appears to take advantage of the future knowledge if and only if it is able to identify and exploit the good channel realizations time slots over the bad ones. As an illustrative example of this phenomenon, we focus on the performance of the zero knowledge, equal-bit and perfect knowledge schedulers in a scenario where all the channel realizations h_r are constant and equal to ϵ, i.e. $\forall t, h_r(t) = \epsilon$. In such a scenario, it is easy to verify that the power strategies p^*_{zk}, p^*_{eb} and p^*_{om} are identical and the performance gap vanishes. This scenario seems to indicate that in the extreme case of a flat channel over the T time slots, the performance gap offered by future knowledge is null. For a performance gap to exist, the channel must be time-varying with significant variations. Note that in this chapter, we do not investigate further how the performance gap might actually depend on the channel variations and leave it for now as it will be investigated more extensively, in next chapter.

3.6 Conclusions, Limits and Future Works

In this chapter, we have investigated how a delay-tolerant network could improve its power-efficiency by taking into account different possible elements of future knowledge. After analyzing theoretically how this future knowledge is exploited by the system to compute the optimal power strategy, we provided numerical results demonstrating the average performance of the system, for several scenarios of future knowledge. The study provided interesting insights:

- The potential performance gain between the optimal perfect knowledge strategy (obtained when perfect knowledge is given) and the worst case scenario (obtained when no knowledge of the future is available) was significant: it then makes sense to look forward to acquiring and exploiting some elements of future knowledge.

- We showed in Section 3.5.4 that the performance gap depends on the time variations of the channel realizations. More specifically, the performance gains depend on the capability of the system of discern good channel realizations from bad channels realizations and exploit them properly.

- Since acquiring a perfect knowledge at $t = 0$ seems unrealistic (even though ideal), we investigated partial and statistical future knowledge schedulers. It turned out that a good statistical knowledge might be sufficient as it allows to approach remarkably the optimal performance bound. Also, acquiring a short-term knowledge, which is realistic, can also enhance the performance of the system.

The presented works could be enhanced by taking into account several possible enhancements that we discuss hereafter:

- **Harnessing the performance gains:** We have provided good insights on the fact that there exist a relation between the channel variations and the performance gap. More work is required in order to harness the exact phenomenon.

- **Power limitation and outage:** In our system model, we considered that the power was not limited by a maximal power p_{max}. However unrealistic, this hypothesis allowed some simplifications. This additional constraint could possibly enhance the performance of the worst-case zero knowledge scheduler, by limiting the amount of power used in the early time slots. It could also limit the power used in the optimal perfect knowledge scheduler, by preventing the system to use a high amount of power on extremely good channel realizations time slots, thus reducing the performance bound. The performance gap would obviously get slightly affected by considering more realistic models for the system, such as a power constraint. Also, if we had considered a power limitation, we should have considered an outage probability constraint during the transmission analysis, as it has been done in [12], which would have complexified a lot the analysis of the optimal strategies.

- **Modulation and Coding Schemes:** Regarding the realistic models altering the potential performance gain, we have also considered that the system could transmit at the channel capacity. a realistic model would instead consider a finite number of Modulation and Coding Schemes (MCS) [134], that could return the transmission rate based on the perceived SINR at the receiver, which obviously depends on both the power strategies and channel realizations.

- **Channel evolution model:** We have considered independent channel realizations in this chapter, but several channel models implement a strong relation between the immediate next channel realization and the present one. Usually, we model such a phenomenon using for example a Markov chain [135] or an auto-regressive model [136]. Considering such a model would probably enhance the prediction made at a present time, about the next channel realization, thus enhancing the prediction capability of the system and, in the end, the performance gains.

- **Different utility function and sleep mode:** In this chapter, we considered an objective function which only take into account the transmission power. We could enhance the power consumption model by considering, for example, a more complete power consumption model, which takes into account the operating and primary costs of an AP, as suggested in [8]. If such a model was considered, less importance would be given to the transmission power costs and we would probably consider scenarios where the AP can be turned into sleep mode, when unused for transmission, which is also a promising feature for power efficiency [13, 14, 15]. It could lead to a new class of strategies, able to take into account sleeping modes, whose investigation could be of interest.

- **Learning about the future from the past channel realizations:** Several papers [36, 37] suggest that we can enhance the predictions made by the system by learning from the previous and present realizations. In a scenario similar to the one described in Section 3.2, where the channel realization followed the same PDFs, we could model a prediction scenario, where the prediction PDFs $D_i^t(h)$ are empirical PDFs, learning from the previous en present realizations, i.e.:

$$\forall t, i, i > t, \hat{D_i^t}(h) = \frac{1}{t}\sum_{k=1}^{t} \delta_{h=h_r(k)} \tag{3.32}$$

  We have decided to not investigate such a class of reactive learning predictors, because we noticed that $\hat{D_i^t}(h) \rightarrow D_{real}^t(h)$ when t goes to infinity. This means that the predictions will become more and more close to the one described in the statistical knowledge scenario, thus leading to a degraded version of the statistical knowledge scheduler performance. However, the study of such learning algorithms might be of interest, especially, in order to acquire a statistical knowledge of the future channel realizations, by learning for the past realizations.

- **Acquiring future knowledge and cost of learning:** We have only focused on the performance gains that the system might get by exploiting elements of future knowledge, but we have not questioned how these elements of future knowledge could be obtained. In particular, we must discuss the 'cost of learning', namely the equivalent power cost required in order to acquire some elements of future knowledge. Investigating this 'cost of knowledge' is a difficult task and still an open question in research at the moment: at the best of our knowledge, there are only a few limited works that are trying to explicit this 'cost of learning'. A few ideas could be found in here [137], even though it is not directly related to wireless networks. More works however focus on defining the 'cost of feedback' [138], namely the cost one has to pay to transmit a piece of information from a central unit in charge of establishing predictions to the AP that needs it. It is a matter of importance, since we need to confront this 'cost of learning' to the potential performance gain that the system could benefit from the acquired future knowledge.

- **Additional requests:** We have also considered that the system does not allow any additional request to enter during the time window $[0, T]$, i.e. that $Q(t)$ can only decrease according to the instantaneous transmission rate. Including possible random arrivals of new requests at the beginning of each time slot and analyze how the optimal strategies tend to adapt, could also be an interesting topic for investigation. Usually, we model the arrival process using Poisson [139], Markov [140] or rational [141] arrival processes and queuing theory. The objective then consists of both transmitting the packets and limiting the outage probability, at deadline T.

- **Extension to multiple users:** Last but not least, we have considered a single user scenario. an extension of the presented work in a multiuser framework with several users transmitting competitively and thus interfering each other would require to complexify the optimization problem. As a matter of fact, and to introduce the next chapter, we investigate such an extension to a multiuser decentralized scenario, where the optimization is selfishly considered at each AP-UE pair. The problem investigated then becomes a non-cooperative multiuser dynamic game.

Chapter 4

A Mean Field Approach to Power-Efficiency in Proactive Delay-Tolerant Transmissions

4.1 Introduction

In this chapter, we consider a downlink delay-tolerant network, similar to the one described in Chapter 3, where a set of Access Points (AP) aim at transmitting several data packets to their assigned Users Equipments (UE), within a predefined time, at a minimal power cost. We assume that the APs are non-cooperating, have perfect a priori knowledge of the future channel realizations and can adapt their transmission powers at will, thus adapting the instantaneous transmission rates to the present channel realizations and interference patterns. We first model the problem as a stochastic multiuser non-cooperative game and recall the complexity of studying a Nash Equilibrium (NE) in a N-body stochastic game. Thanks to symmetries between users, and assuming a large population of homogeneous users, we transition our problem into a Mean Field Game (MFG), with tractable fundamental equations. Transitioning into a MFG allows us to bypass the mathematical complexity of a multiuser stochastic game, with any number of users N, by representing it with an equivalent game whose complexity is lower, since it consists of a 2-body problem. The presented framework yields an analysis of the mean field equilibrium and optimal transmission power strategies, which allows every AP to, selfishly but rationally, satisfy its transmission needs, at a reduced power cost, compared to classical (full-power or constant) power strategies, which are unable to exploit the latency and/or of the future knowledge offered to the system.

The remainder of this chapter is structured as follows. After introducing the motivations, related works and contributions, we detail in Section 4.2 the system model considered for the delay-tolerant network and define the non-cooperative multiuser stochastic game, to be studied throughout this chapter. In Section 4.3, we provide elements of analysis showing the inherent mathematical complexity of investigating a multiuser discrete non-cooperative game, thus showing the limits of the classical analysis. In Section 4.4, we define simple reference strategies that are used for performance comparison with the optimal power strategies in simulations. In Section 4.5, we present a short tutorial on the Mean Field Theory, that allows us to simplify the stochastic game previously detailed, by transitioning it into a MFG. We conduct the analysis of the Mean Field Equilibrium for the MFG and explicit the fundamental two equations that rule the equilibrium, namely the Hamilton-Jacobi Bellman equation (HJB) and the Fokker-Planck-Kolmogorov equation (FPK). Once established, we detail an iterative method for approaching the equilibrium and provide a numerical method, based on finite differences, that allows to numerically compute the HJB and FPK equations separately. The following four sections present numerical results for different scenarios of channel models, with a progression in terms of complexity. In Section 4.6, the channels are all constants and equal to 1. In Section 4.7, the channels are constant wrt. time, but their value may vary from one AP-UE pair to another. In Section 4.8, the channels are now time-varying and their evolutions follow an auto-regressive model of order 1, whose parameters a perfectly known. In Section 4.9, the channel models are defined according to a stochastic auto-regressive model, used for modeling uncertainty, estimation errors,etc. In every scenario, we detail the fundamental equations to be solved for characterizing the MFG equilibrium, and compare the performance of the optimal strategies obtained via this equilibrium to the performance of the reference strategies detailed in Section 4.4. Finally, Section 4.10 concludes the chapter, by summing up the contributions, discussing the limits of the presented work and future works.

4.1.1 Motivations and Related Works

With the increasing trend for higher data rates, network operators are required to provide higher and higher Quality of Service (QoS) to their customers, while reducing at the same time their operational costs [1]. In that sense, and thanks to the new paradigms of cognitive radios [7], networks are now able to control and adapt their transmission parameters to the present environment and transmission context [142]. It immediately leads to the concept of green power-efficient networks, a framework where the network can adapt the transmission power of its equipment to the present transmission needs and context, in order to reduce the global power consumption of the network [143]. The study of power control problems has then become a relevant issue and a promising challenge for multiuser communications and green power-efficient networking, with multiple articles in literature [144, 145, 71, 10].

Moreover, recent studies have revealed that the usage of the wireless devices and network by human was highly and accurately predictable [29]. It appeared that it is becoming more and more conceivable to foresee the whereabouts, mobilities and future transmission contexts of human individuals and their equipments [28]. Exploiting such a future knowledge can for example be used in order to anticipate outage situations, by smartly pre-buffing videos, as in [38, 39]. Assuming a priori knowledge about future transmission contexts can be accessed, the classical latency vs. energy efficiency trade-off has also shown great interest, heading to the so called concepts of delay-tolerant networks [20, 22, 21]. In such delay-tolerant networks, the transmission are not urged, so that the rate constraint (typically completing a given transmission on a given time window) is not satisfied as soon as possible. Instead, the network is only required to complete its transmission before a given deadline, thus allowing the system to smartly schedule its transmissions and adapt the transmission settings to the upcoming transmission context, for which it has perfect prior knowledge.

In that sense, we propose, within this chapter, to study a decentralized power control problem, as it was originally introduced by Goodman and Mandayam in [146] or more recently in [147]. In our problem, every user is endowed the capability of selfishly adapting its transmission settings (powers and rates), depending on its own rate constraint, its prior knowledge of the future transmission context (network, channels dynamics, etc.). We model the prior knowledge, by considering that the system has knowledge of the parameters used for modeling the channel dynamics according to a stochastic auto-regressive (AR) model [51, 148]. As a rate constraint, we consider that each AP-UE pair must complete a given data packet transmission within a deadline. The users are also sharing common resources (bandwidth, time spectral resources, etc.) and compete when transmitting. The competition between users is expressed through interference, due to other users attempting to transmit at the same time on an adjacent cell. The decentralized power control problem is then naturally formulated as a multiuser non-cooperative stochastic game, [53]: each user seeks its optimal transmission powers strategy, i.e. the strategy that minimizes its utility function, which consists of the total power consumption, while ensuring a complete transmission of a data packet of initial size known, within a predefined deadline.

In mathematical terminology, the set of power strategies that will minimize the considered utility function corresponds to a Nash Equilibrium (NE) [54] of the stochastic non-cooperative game. When the NE is reached, no user wishes to deviate independently from its own power strategy, since any deviation would lead in the end, to a worse utility for this user (incomplete transmission or, higher power consumption). Studying NE in such a context is then relevant. In such multiuser stochastic games, it is possible to prove the existence of a NE and to define sets of N Partial Differential Equations (PDE), namely the Hamilton-Jacobi-Bellman (HJB) equations, one for every single user of the system [53, 149, 150]. However, solving these sets of equations, in order to characterize the NE of the game, becomes complicated and even impossible, when the number of AP-UE pairs N grows large (especially when $N > 2$).

Nevertheless, it appears that our problem has a well-designed structure that presents symmetries among the competitive users. It is then possible to simplify the problem, by exploiting those symmetries when the number of users N grows large [62]. The Mean Field Theory allows to approximate a multiuser stochastic game and turn a N users game into a more tractable equivalent game, called a Mean Field Game (MFG) [63, 64, 65]. Several recent papers have implemented such a Mean Field framework, in order to simplify the resolution of multiuser stochastic games. For example, in [66, 67], every user has to adapt their strategies to the quality of their environment(link quality, channel, etc.), while ensuring a minimal SINR constraint. In [68], a similar and interesting analysis is provided, with an application of the MFG tools, into the topic of electrical vehicles in the smart grids. In [69, 61], the players are transmitters, who adapt their transmission powers to the quality of their link with the receiver, the strategies of the other users, and their battery level, while ensuring a SINR constraint. In a similar way, we turn an untractable N users stochastic game into a MFG and study the Mean Field Equilibrium of the new-built game. The Mean Field Equilibrium leads to the mean field optimal set of power strategies, that will be used for approximate the optimal strategies of the original N users stochastic game.

Finally, we study and analyze the optimal strategies and challenge their performance, in term of total power consumption, compared to two reference strategies. In the first reference strategy, the system transmits using maximal transmission power until the transmission is completed and then stops transmitting once complete. This reference scenario is used to model the power consumption of a transmission strategy, which transmits notably in advance to complete a given transmission as soon as possible. Such a transmitter is unable to exploit the offered latency to the system. In the second transmission strategy, the system can define a constant power level that will be used for transmission on all the time slots. By doing so, the system is able to exploit the offered latency, but is unable to adapt the transmission power to the present and future context. More specifically, it is unable to adapt the powers to the good or bad channel realizations, as a time water-filling algorithm would [48, 49, 50]. We study the performance of each power strategy, for several channel models (from the constant channel case to the complete stochastic problem) and we provide numerical simulations assessing the performance of the investigated optimal MFG and reference strategies.

4.1.2 Contributions

The content of this chapter has been published in three papers. In the first conference paper, we introduce the system model and define the two fundamental PDE for analyzing the equilibrium in the Mean Field Game, simulations results are also provided for the constant channel scenario [114] . In the two following papers, one journal and one conference paper, the complete analysis and numerical results for both time varying channels and stochastic scenarios are provided [113, 119]. The innovation and scientific contributions presented in this chapter are summed up as follows:

- First, we propose a delay-tolerant transmission model capable of exploiting a perfect future knowledge, in a network composed of N AP-UE pairs, whose objectives consists of transmitting a given data packet, within a predefined deadline T, at a minimal power cost. The objective for each AP-UE pair is to define, selfishly but rationally, the optimal strategy that ensures the completion of the given transmission constraint, at a minimal power cost. Our first contribution relies on defining the system model representing this non-cooperative delay-tolerant transmission problem with N AP-UE pairs. Mathematically speaking, the problem consists of a multiuser non-cooperative stochastic game. After providing a first analysis of the discrete case, we highlight the inherent mathematical complexity of studying a Nash Equilibrium in such a game.

- When faced with this mathematical complexity, which renders the problem completely untractable when the number of users N grows larger than 2, we propose to exploit the recent Mean Field Theory works: we transition our initial problem into a stochastic Mean Field Game, with reduced mathematical complexity. The analysis of the new-built game is then conducted, leading to a set of two fundamental equations, namely the Hamilton-Jacobi-Bellman (HJB) and the Fokker-Planck-Kolmogorov (FPK) equations. These equations allow to define and compute the optimal power strategies to be used by any user of the system, in any configuration.

- We also detail an iterative algorithm that can be used for approaching the Mean Field Equilibrium of the Mean Field Game. We also suggest a numerical procedure, based on the finite differences method, allowing to numerically compute the two fundamental equations separately. Both the iterative procedure and equation solvers described in this chapter can be reused and adapted to any stochastic Mean Field Game.

- Finally, we compare the performance of the optimal power strategy returned by the MFG and compare it to a set of two reference power strategies. By doing so, we are able to provide good insights on the potential performance gains that are offered by such a delay-tolerant transmission framework, when coupled with perfect future knowledge.

4.2 System Model and Optimization Problem

4.2.1 System Model

In this chapter, we consider a downlink narrow-band system with N Access Points (AP) and N User Equipments (UE), represented in Figure 4.1. We assume a one for one assignment, i.e. $\forall i \in \mathcal{N} = \{1, ..., N\}$, UE i is assigned to AP i. The present model does not consider any handover procedure, nor does it allow Coordinated Multi-Point (CoMP): the UEs remain assigned to their AP during the whole simulation. Although the model can be enhanced to take into account such procedure, we stick to this model, throughout the chapter, for simplicity's sake. We discuss the extension of this work, with CoMP and/or handover in Section 4.10. We denote $H(t)$ the $N \times N$ channel matrix at time slot t, whose general term $h_{ij}(t)$ denotes the channel realization at time slot t between AP i and UE j. Moreover, we assume that the channel realizations are in a bounded set $\mathcal{H} = [h_{min}, h_{max}]$, with $0 < h_{min} < h_{max} < \infty$. We assume block-fading quasi static channels [46, 151, 152]: each channel realization $h_{ij}(t)$ remains constant during the whole duration Δt of the time slot $t \in \mathcal{T} = \{1, ..., T\}$. The channel realizations, can however be time-varying. To model the dynamics of the channels, we consider that they follow an Itô process [51], as explicated in [153]. The channel dynamics are then modeled as follows:

$$\forall t, dh_{ij}(t) = \alpha_{ij}(t, h_{ij}(t))dt + \sigma_b(t)dW_{ij}(t) \tag{4.1}$$

Where

- α_{ij} is a smooth function, known,
- $dW_{ij}(t)$ are mutually independent Wiener processes with variance $\sigma_b(t)$,
- the initial channel realization $h_{ij}(0)$ are known, i.e. $H(0)$ is known.

In discrete time, this rewrites:

$$\forall t, h_{ij}(t+1) = h_{ij}(t) + \alpha_{ij}(t, h_{ij}(t))\Delta_t + \sigma_b(t)dW_{ij}(t) \tag{4.2}$$

According to [149], the following channel dynamics definition is sufficient and leads to a unique trajectory for the channels $h_{ij} = (h_{ij}(t))_{t\in\mathcal{T}}$. The deterministic par $\alpha_{ij}(t)$ allows to take into account both the path loss and the shadowing effects. Such a model can be used to model the channel evolution due to the mobility of the user. The stochastic part, on the other hand can be used to model the rapid and unpredictable variations of the channels, as well as the channel prediction and/or estimation uncertainty.

We consider a delay-tolerant network, where every AP i is required to transmit a given data packet of initial size $Q_i(0)$ to its assigned UE i, over a given time window of T time slots of duration Δt. In the following, we will denote $Q_i(t)$ the remaining packet size that still has to be transmitted at the end of time slot t, $\forall t \in \mathcal{T}$. In particular, $Q_i(0) > 0$ denotes the initial packet size, which is known at the beginning of the first time slot. Each AP can freely adapt, at the beginning of each time slot, the transmission power $p_i(t)$ to be used during the whole time slot t. By doing so, each AP can then adapt the instantaneous transmission rate to be used during the present time slot t. More specifically, we assume that the packet size decreases according to the achievable information rate, when interference caused by the transmissions of the other users are treated as an additive source of noise. This means that the remaining packet size $Q_i(t)$ decreases according to:

$$dQ_i(t) = -\omega_i(t, X, p)dt = -Blog_2\left(1+\gamma_i(t)\right)dt \tag{4.3}$$

Which rewrites in discrete time, as:

$$Q_i(t) = Q_i(t-1) - \omega_i(t, X, p)\Delta_t = Q_i(t-1) - Blog_2\left(1+\gamma_i(t)\right)\Delta_t \tag{4.4}$$

And γ_i is the perceived SINR at receiver i and time t, which depends on the transmission powers used by every AP of the system and the channel realizations, through:

$$\forall i, \forall t, \gamma_i(t) = \frac{p_i(t)h_{ii}(t)}{\sigma_n^2 + I_i(t)} \tag{4.5}$$

Where σ_n^2 is the variance of the noise realizations, which are assumed to be iid realizations of a centered Gaussian process with variance σ_n^2, and $I_i(t)$ is the interference perceived by UE i during time slot t, due to concurrent transmissions from other APs, when interference is treated as noise. Here, the interference consists of the sum of all the other users contributions and has been normalized. Several applications can justify this normalization: for example, in Code Division Multiple Access (CDMA) with random spreading systems [154, 155, 156]. Later on, in Section 4.5, this hypothesis will be revealed as a necessary assumption in order to transition into a Mean Field Game. The interference term can then be expressed as:

$$\forall i, \forall t, I_i(t) = \frac{1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^{N} h_{ji}(t) p_j(t) \tag{4.6}$$

A complete transmission for the AP-UE pair i then verifies:

$$Q_i(T) = Q_i(0) - \sum_{t=1}^{T} \omega_i(t, X, p) \Delta_t = Q_i(0) - \sum_{t=1}^{T} B log_2 \left(1 + \gamma_i(t)\right) \Delta_t = 0 \tag{4.7}$$

Finally, we denote $\mathcal{Q} = [0, \max_i (Q_i(0))]$, the set of possible values for any remaining packet size $Q_i(t)$, for any AP-UE pair i and any time slot t. We sum up all the presented concepts in two figures: Figure 4.1 give a schematic representation of the system to be considered, whereas Figure 4.2 provides an illustrative diagram, about the transmission concepts.

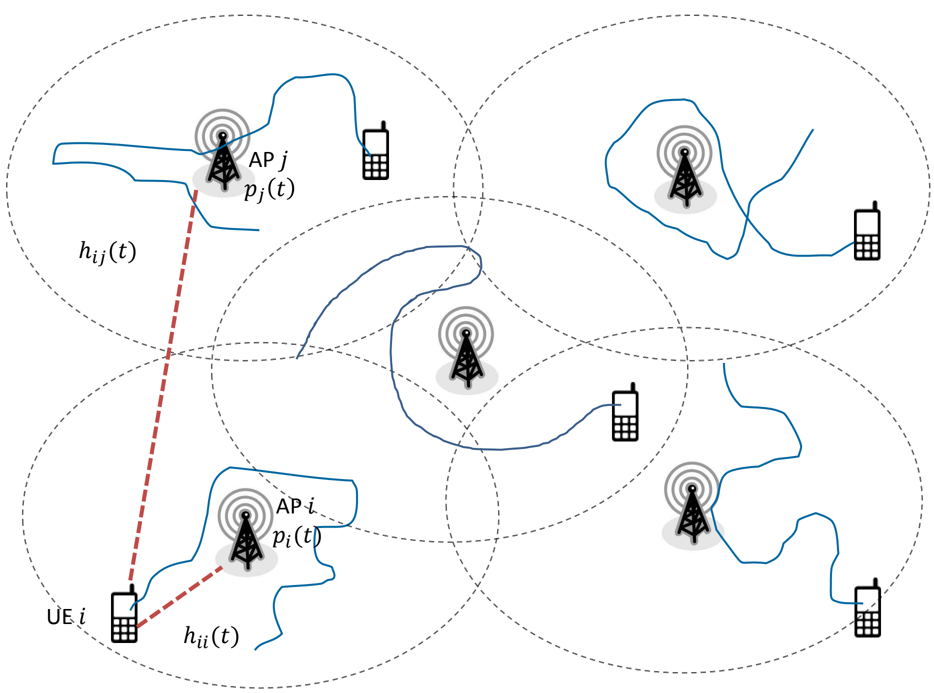

Figure 4.1: System Model: N AP-UE pairs with mobile users, time-varying channels.

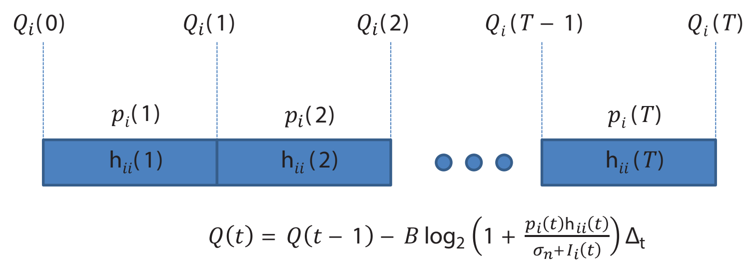

Figure 4.2: Transmission diagram for user i, by analogy from the previous Figure 3.1

4.2.2 The multiuser Non-Cooperative Stochastic Game

In this chapter, the objective is to define, for each AP-UE pair i, the optimal power strategy $p_i^* = (p_i^*(1), ..., p_i^*(T))$, with $\forall i, t, p_i^*(t) \geq 0$, which allows each AP i to transmit completely its packet of initial size $Q_i(0)$, before the deadline of T time slots, at a minimal power cost. Each AP-UE pair i must then define, at the same time and at the beginning of each time slot, the optimal power strategy $p_i^*(t)$, to use during time slot t, based on the revealed present channel realizations, the packet size remaining and the future knowledge. Each AP i is then confronted to the following optimization problem (4.8), at the beginning of each time slot $t \in \mathcal{T}$:

$$\begin{array}{l} p_i^*(t) = p_i^*(t) \in (p_i^*(u))_{u\in[t,T]} \\ p_i^*(u)_{u\in[t,T]} = \arg\min_{(p_i(u))_{u\in[t,T]}} \left[\mathbb{E}\left[\sum_{u=t}^{T} p_i(u) + K(Q_i(T))\right]\right] \\ \text{s.t. } \forall u \in [t,T], dX(u) = f(u)dt + F(u)dW(u) \end{array} \tag{4.8}$$

With initial conditions $X(t)$ known

- $K(Q_i(T)$ denotes the penalty function, based on the remaining packet size at the end of the last time slot T, $Q_i(T)$. This final penalty function is used to relax the final constraint, defined by equation (4.7). This penalty function must ensure that the system will transmit in order to avoid the penalty of not completing a given transmission before the deadline. Basically, it does not penalize the system if the transmission has been completed, i.e. $K(Q(T)) = 0$ if $Q(T) \leq 0$, and it heavily penalizes the system if the transmission is incomplete, i.e. $Q(T) > 0$. More details about this penalty function to be considered can be found in Sections 4.6.

- $X(u)$ is the present state of the system at time slot t, i.e. the following column vector:

$$\forall t, X(u) \text{ is the column vector } [Q_1(u-1), ..., Q_N(u-1), h_{11}(u), h_{12}(u), ..., h_{NN}(u)] \tag{4.9}$$

- $\forall u \in [t,T]$, $f(u)$ denotes the following column vector, used for modeling the deterministic part of the system dynamics:

$$f(u) = \begin{pmatrix} -Blog_2\left(1+\gamma_1(u)\right) \\ \vdots \\ -Blog_2\left(1+\gamma_1(u)\right) \\ \alpha_{11}(u) \\ \alpha_{12}(u) \\ \vdots \\ \alpha_{NN}(u) \end{pmatrix} = \begin{pmatrix} -\omega_1(u,X,p) \\ \vdots \\ -\omega_N(u,X,p) \\ \alpha_{11}(u) \\ \alpha_{12}(u) \\ \vdots \\ \alpha_{NN}(u) \end{pmatrix} \tag{4.10}$$

- F(u) is the $N(N+1) \times N(N+1)$ matrix defined by block as:

$$\forall t, F(u) = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_b(u)^2 I_{N^2} \end{pmatrix} \tag{4.11}$$

- Where I_{N^2} is the identity matrix of size $N^2 \times N^2$ and $dW(u)$ refers to a $N(N+1)$ column vector of independent Wiener processes, defined as $dW(u) = \left(\underbrace{0, ..., 0}_{N \text{ times}}, dW_{11}(u), dW_{12}(u), ..., dW_{NN}(u)\right)$. The $dW_{ij}(u)$ elements are mutually independent Wiener processes with variance $\sigma_b(u)$.

Assuming the initial packet sizes $Q(0) = (Q_1(0), ..., Q_N(0))$ and the initial channel states H(0) are known, (4.8) is a well-defined N-users non-cooperative stochastic game [53].

4.3 Analysis of the Game Equilibria

In this section, we focus on the analysis of the Nash Equilibrium for the N-users stochastic game (4.8) and show the inherent mathematical complexity of such an analysis when the number of users N becomes larger than 2. Let us first denote $C_i(t,p) = \mathbb{E}\left[\sum_{u=t}^{T} p_i(u) + K(Q_i(T))\right]$, the utility function perceived by the AP-UE i, when the set of played strategies are $p = (p_i(u))_{u\in[t,T]}$. We provide first, the definition of a Nash Equilibrium for game (4.8).

Proposition 4.1. *A given power strategies set $p^* = (p_i^*(u))_{u\in[t,T]}$ is a Nash Equilibrium [54] for game (4.8) if and only if:*

$$\forall i \in \mathcal{N}, \forall p_i, C_i(t,p^*) \leq C_i(t,p_{-i}^*) \tag{4.12}$$

Where $p_{-i}^* = \left(p_1^*, ..., p_{i-1}^*, p_i, p_{i+1}^*, ..., p_N^*\right)$.

Analyzing Nash Equilibria in such a context is a matter of interest: we show that the optimal power strategy $p^* = (p_1^*, ...p_N^*)$ is by definition a Nash Equilibrium, since no AP-UE pair will independently deviate from its strategy p_i^*, when the Nash Equilibrium is reached, as it might lead to a worst configuration (either an incomplete transmission, which will be penalized by the penalty function or a higher power consumption). This also means that p_i^* is the best power strategy to be implemented by AP-UE pair i, when the initial state is $X(0)$ and other AP-UE pairs j ($j \neq i$) from the system implement power strategies p_j^*. In that sense, p_i is the best - and thus a satisfying - strategy for the AP-UE i when the initial state is $X(0)$ and the other users implements strategies p_j^*. Our first concern before investigating a possible Nash Equilibrium of the game, is to ensure that such an equilibrium exists. As detailed in Proposition 4.2, a Nash Equilibrium for the game (4.8) exists.

Proposition 4.2. *For each AP-UE pair i,*

- *$p_i(t)$ takes values in a compact and convex set,*
- *the utility function is continuous,*
- *the utility function is concave for any $p_i(t), t \in \mathcal{T}, i \in \mathcal{N}$, when the powers used by different users $j \neq i$ or the powers on different time slots $t' \neq t$ are fixed.*

Based on [57, 53], there exist a Nash Equilibrium for the game. Also, thanks to [157], the unicity of the Nash Equilibrium can also be proven.

It is easy to affirm that our game is well-designed and verifies the three listed hypotheses, as long as our penalty function K is smooth. In order to analyze the Nash Equilibrium of game (4.8), let us first denote $v_i(t, X, p)$ the running cost function or Bellman function [158] for the AP-UE i, as:

$$v_i(t, X, p) = \mathbb{E}\left[\sum_{u=t}^{T} p_i(u) + K(Q_i(T))\right] \tag{4.13}$$

The Bellman function models the expected power cost for user i, when the present time is t, the state at time t is X, and the power strategy to be used for all the users are $p = (p_i(u))_{i\in\mathcal{N}, u\in[t,T]}$. Intuitively, we observe that the running cost function can be decomposed in two parts:

- The first one refers to the expected remaining power cost between the present time t and the deadline T, if the present state at time t is X and the AP-UE pair i implements the power strategy $p_i(u), u \in [t, T]$.

- The second part refers to the penalty function, eventually given to the AP-UE pair i if the power strategy set p does not allow a complete transmission before deadline T for AP-UE pair i (i.e. $Q_i(T) > 0$)

The objective consists of finding power strategies $p^* = (p_1^*, ..., p_N^*)$ such that p^* is a Nash Equilibrium. The Nash Equilibrium can, in fact, be characterized using the running cost functions v_i.

Proposition 4.3. *Since (4.8) is well-defined, according to [159, 53], there exist a unique Nash Equilibrium p^* ([157]) and the related running cost functions $v_i^*(t, X) = v_i(t, X, p^*)$ verify the Hamilton-Jacobi-Bellman (HJB) equations, applied to each AP-UE pair i, defined as:*

$$\begin{aligned} &\min\nolimits_{p_i(t)} \Big[p_i(t) + \textstyle\sum_{j=1}^{N}\sum_{k=1}^{N} \alpha_{jk}(t)\partial_{h_{jk}} v_i^* - \sum_{j=1}^{N} \omega_j(t, X, p)\partial_{Q_j} v_i^* \\ &+ \tfrac{1}{2}\textstyle\sum_{j=1}^{N}\sum_{k=1}^{N} \sigma_b^2 \partial^2_{(h_{jk}h_{jk})} v_i^*\Big] + \partial_t v_i^* = 0 \end{aligned} \tag{4.14}$$

Moreover, the optimal power strategy p_i^* verifies the infimum term, i.e. $\forall i, \forall t$:

$$p_i^*(t) = \arg\min_{p_i(t)} \left[p_i(t) + \sum_{j=1}^{N}\sum_{k=1}^{N} \alpha_{jk}(t)\partial_{h_{jk}} v_i^* - \sum_{j=1}^{N} \omega_j(t, X, p)\partial_{Q_j} v_i^* + \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N} \sigma_b^2 \partial^2_{(h_{jk}h_{jk})} v_i^*\right] \tag{4.15}$$

Proposition 4.4. *The instantaneous power strategies $p^*(t) = (p_1^*(t), ..., p_N^*(t))$ to be used at any time t are defined as solutions to the following set of N*

polynomial equations, $\forall i \in \mathcal{N}$*:*

$$1+\frac{B}{log(2)}\frac{h_{ii}(t)}{\sigma_n^2+\frac{1}{N-1}\sum_{\substack{j=1\\j\neq i}}^{N}p_j(t)h_{ji}(t)+h_{ii}(t)p_i(t)}\partial_{Q_i}v_i^*(t,X)$$
$$+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{B}{log(2)}\frac{p_j(t)h_{jj}(t)h_{ij}(t)}{(\sigma_n^2+\frac{1}{N-1}\sum\limits_{\substack{k=1\\k\neq j}}^{N}p_k(t)h_{kj}(t))(\sigma_n^2+\frac{1}{N-1}\sum\limits_{\substack{k=1\\k\neq j}}^{N}p_k(t)h_{kj}(t)+h_{jj}(t)p_j(t))}\partial_{Q_j}v_i^*=0 \tag{4.16}$$

Proof. Refer to Appendix 8.3 for elements of proof. □

The instantaneous power strategies are then defined according to a set of N polynomial equations of order $(2N-1)$. In order to find $p_i^*(t)$, we must solve the set of equations, with $(p_j(t)^*)_{j\in\mathcal{N}}$ as unknowns variables. Doing so, is complicated, even for $N=2$. When it can be computed theoretically, the $p_i^*(t)$ closed-form expression is usually reused in the min term from the HJB equation (4.14), in order to obtain a set of N coupled HJB equations that characterize the Nash Equilibrium of the game. The set of N HJB equations is hard to solve, even for $N=3$, because i) the HJB are non linear and ii) the number of partial derivatives in each equation tends to skyrocket with N: each HJB includes partial derivatives wrt $h_{ij},(i,j)\in\mathcal{N}^2$ and $Q_i, i\in\mathcal{N}$.

4.3.1 An Approximation of the Dynamic Game Equilibrium

We have demonstrated in the previous section, that in order to characterize the Nash Equilibrium of the game, we must solve a set of N polynomial equations and a set of N coupled PDEs, namely HJB equations. At this moment, we have been facing two major difficulties:

- First, it appears complicated to compute the closed-form expression of the optimal instantaneous power to be used by any AP-UE pair i on any time slot t, $p_i^*(t)$, as it requires to solve a set of N polynomial coupled equations in $(p_j(t))_{j\in\mathcal{N}}$.
- If we were able to express the closed-form expression of $p_i^*(t)$, expressed as a function of the system parameters and running cost functions $(v_j^*(t,X))_{j\in\mathcal{N}}$, we would have to reuse this expression in the min term of the N HJB equations (4.15). The objective would then be to solve a set of N coupled PDEs in $(v_j(t,X))_{j\in\mathcal{N}}$, which is complex, even in a $N=2$ scenario.

In order to tackle this mathematical complexity and compute the Nash Equilibrium of the game, a classical approach, when no stochasticity exists (i.e. $\sigma_b = 0$) relies on the iterative time water-filling algorithm, with several examples in literature [56, 57, 58, 59], as follows. The core idea is to focus on a every single AP-UE pair i and realize that:

- The expression of the competition between a AP-UE pair i and the other pairs $j \neq i$ consists only of the interference pattern $(I_i(t))_{t\in\mathcal{T}}$, which is a weighted sum of the other pairs power strategies p_j.
- When the interference pattern for this user i, denoted $(I_i(t))_{t\in\mathcal{T}}$ is known, then its optimal power strategy in response to this interference pattern, can be analytically computed and is defined according to a simple time water-filling algorithm, similar to the one previously detailed in Section 3.4.1.

More specifically, if we focus on a single AP-UE pair i and assume that the other pairs $j \neq i$ have fixed power strategies p_j, thus generating an interference pattern for pair i, denoted $I_i(t)$ and computed as equation (4.6), then we can compute the optimal power strategy p_i^*, which minimizes the objective function, taking into account the transmission completeness and the total power cost. The considered optimization problem is then defined as follows:

$$\begin{aligned}&p_i^* = \arg\min_{p_i}\left[\textstyle\sum_{t=1}^{T} p_i(t) + K(Q_i(T))\right]\\ &\text{s.t. } dX(t) = f(t)dt + F(t)dW(t) \text{ defined as in equation (4.8)}\end{aligned} \tag{4.17}$$

And the optimal power strategy p_i^*, to be used by the AP-UE pair i in response to the interference pattern $I_i(t)$ is defined as:

$$\forall t \in \mathcal{T}, p_i^*(t) = \left(\mu - \frac{\sigma_n^2 + I_i(t)}{h_{ii}(t)}\right)^+ \tag{4.18}$$

Where μ is the unique water level verifying $\mu = \arg\min_{\mu}\left[\sum_{t=1}^{T} p_i(t) + K(Q_i(T))\right]$, that can be computed using a dichotomic search algorithm. Also, the notation $(x)^+$ refers to $(x)^+ = \max(x, 0)$. Assuming we can compute the optimal strategy to be used by any AP-UE pair i in response to the interference pattern $I_i(t)$, generated by the other power strategies picked by the other pairs $j \neq i$, the unique Nash Equilibrium is often computed as the unique fixed point of the iterative algorithm, detailed hereafter in Algorithm 1.

Data: Present data packets $Q_i(0)$, and channel dynamics $H(t)$
Result: The Nash Equilibrium configuration $p^* = (p_1, ..., p_N)$
Initialize all the power strategies as 0, i.e. $\forall t, i, p_i(t) = 0$;
Initialize all the interference patterns 0, i.e. $\forall t, i, I_i(t) = 0$;
while *a convergence criterion on* $p = (p_1, ..., p_N)$ *is not satisfied* **do**
    For $k = 1, ..., N$ Update p_k as the time water-filling response to interference pattern I_k;
    Update the interference patterns for other pairs I_j, $j \neq k$;
end
The fixed point of the algorithm is the Nash Equilibrium of the game.;

Algorithm 1: The iterative time water-filling algorithm for the N-users non-cooperative dynamic game.

Basically, the iterative algorithm selects each single pair in order and allows it to update its power strategy to the present interference pattern. When the power strategy of a single user is modified, the interference patterns perceived by all the other users of the system changes and the previous power strategy is not necessarily optimal anymore. This phenomenon is classically called the 'ping pong effect' and is illustrated in Figure 4.3. The iterative process is repeated until a convergence is observed on $p = (p_1, ..., p_N)$. The objective is then to approach the fixed point configuration p^* of the iterative algorithm, which by definition is the Nash Equilibrium, as it consists of a configuration where no pair i wishes to deviate independently from its present power strategy p_i^*. For such an algorithm, it can be proven that it converges to the fixed point and Nash Equilibrium configuration p^*, no matter what the initial configuration is, as in [10]. However, the computation time for approaching the fixed point p^*, which depends on the precision (required in the convergence criterion used in the while loop of the algorithm), explodes when the number of AP-UE pairs N in the system becomes large, thus rendering the problem rapidly untractable when the number of AP-UE pairs N becomes larger than 2. Two solutions, then: use a simple suboptimal heuristic power strategy, as in Section 4.4 or transition into a Mean Field Game, as detailed in Section 4.5.

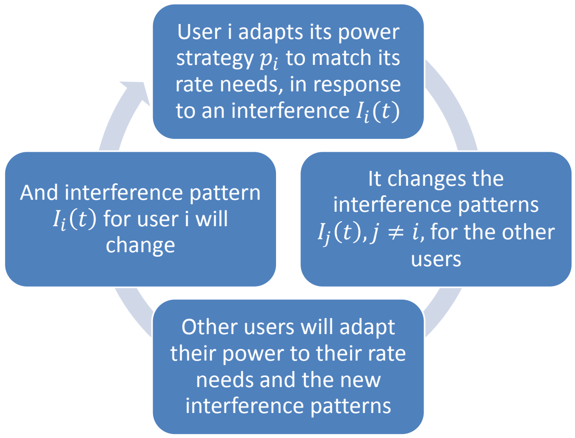

Figure 4.3: Illustrative example of the 'ping pong effect' phenomenon.

4.4 Additional Reference Strategies

4.4.1 Constant Power Strategies

A simple way to help the iterative process convergence consists of setting an additional constraint on the power strategies. In this section, we assume that every single AP-UE pair i can adapt its power strategy p_i at each iteration of the Algorithm 1, but it can only define a single power level μ_i, that will be used on every single time slot for transmission, i.e.:

$$\forall i, \forall t, p_i(t) = \mu_i \geq 0 \tag{4.19}$$

In the following, we will denote $p^{cst} = (p_1^{cst}, ..., p_N^{cst})$ the fixed point strategy that is returned by the iterative Algorithm 1, when this additional constant power constraint is considered. Such a strategy is able to take into account the latency of T time slots offered to the system, but is unable to exploit the offered prior knowledge of the future channel realizations, by adapting the power level to the good or poor channel realizations. In that sense, it is quite similar to the equal-bit scheduler we have described in Section 3.4.3.

4.4.2 Full-power Strategies

Another simple strategy that will be considered as reference, is the strategy which tends to satisfy the transmission as soon as possible. In order to model this strategy, we will consider that the system transmits at a high power P_{high}, at the beginning, and transmits using this power until the packet transmission is complete. The power strategy, denoted p_i^{rush} is defined for any AP-UE pair i, as:

$$\forall t \in \mathcal{T}, p_i^{rush}(t) = \begin{cases} P_{high} & \text{if } Q_i(t-1) > 0 \\ 0 & \text{else.} \end{cases} \tag{4.20}$$

The packet size evolution for each user i, $Q_i(t)$ follows:

$$Q_i(t) = Q_i(t-1) - Blog_2\left(1+\gamma_i(t)\right)\Delta_t \tag{4.21}$$

Where

$$\gamma_i(t) = \frac{p_i^{rush}(t) h_{ii}(t)}{\sigma_n^2 + \sum_{\substack{j=1 \\ j\neq i}}^{N} h_{ji}(t) p_j^{rush}(t)} \tag{4.22}$$

This strategy is unable to exploit either the offered latency nor the future knowledge provided to the system. Instead, it models the cost of an instantaneous transmission in a context where every AP wishes to rush its transmission, in order to complete it as soon as possible. In numerical simulations, we consider that the power level P_{high} is equal to 5 times the maximal value of the constant power strategy of any user, i.e. $P_{high} = \max_i [\mu_i]$.

4.5 Transitioning into a Mean Field Game

4.5.1 Defining an Equivalent Mean Field Game

We have shown in the previous Section 4.3 that computing the Nash Equilibrium for the N-users non-cooperative stochastic game, was complicated, especially when the number of AP-UE pairs N becomes large or when stochasticity is considered. However, our system has a particular structure in the sense that it presents symmetries between users. Under these assumptions, and if we assume the number of players in the game N is large enough to be considered infinite, the Mean Field theory can come into play: the multiuser stochastic non-cooperative game can be reformulated as a Mean Field Game, the limit game of the previous game (4.8) when N goes to infinity. Whatever the initial number of users N in the system, we transition to an equivalent game with only two bodies. as a consequence, the Mean Field Equilibrium, which is the equivalent of the Nash Equilibrium in a Mean Field Game, can then be characterized by only two tractable equations. Initially introduced by Lasry and Lions, in [63, 64, 65, 62], the general framework of Mean Field theory relies on the following four hypotheses:

- **Rational expectations and behaviors of the players:** The first hypothesis was described by Muth [160] and is now commonly accepted in game theory. It basically means that the users decisions are rational and are based on the utility functions. Players anticipate the evolution of the overall state of the system to define their strategy and minimize their utility functions. In our game, it is strictly equivalent to players adapting their present power strategy, by taking into account both the current state (t, X) and the future knowledge about the channel evolution.

- **Players anonymity:** The second crucial hypothesis is the essence of Mean Field Theory. Basically, it states that the players can be anonymized: the optimal strategy $p_i^*(t)$ of a player i depends only on its state (t, X) and its perception of the $N-1$ other actions, which was initially modeled through interference. As a consequence, any permutation of two players will not change the outcome of the game. This hypothesis is commonly referred to as the indistinguishability property [66]: two users sharing an identical state will implement the same optimal power strategy. Based on this hypothesis, we can define a unique set of controls $p(t, X)$, the mean field power strategy, which relies only on the player state (t, X) and that applies to every single player of the system. As a consequence, we have already simplified the game, since we must now define only one set of power strategies to be used by every player in any state, instead of N individual strategies. In the following, we can then drop the index i denoting the users, as they can be considered anonymous and share the same mean field power strategy $p(t, X)$.

- **Continuum of a large number of players:** The third hypothesis is based on the assumption that the number of users N is large enough to be considered infinite, and this large population of users can then be modeled as a continuum of users. When coupled with the second hypothesis, this means that we can model the state of the N users at any time, by using a a distribution of players $m(t, X)$ based on their states. We detail this distribution of users later on.

- **The social interaction between players can be described through a mean field:** The fourth hypothesis, however, is absolutely specific to the mean field games and relies on the way the interactions between players are modeled. It requires that the interaction between one user and the others can be based on the empirical distribution of all the states of the players, which is the case in our system: from a single user point of view, the interaction with the $N-1$ other users does not consist of one-to-one interactions, instead the user is affected by a joint response of the $N-1$ users altogether, namely the perceived interference at our user receiver side. In our previous description, the interference term initially consisted of a weighted sum of the power strategies of the $N-1$ other users. Assuming the users in the system are anonymized and implement the same power strategy $p(t, X)$, this term can be rewritten, so that the interference is expressed using the power strategy $p(t, X)$ and the current state of the system $m(t, X)$. As a consequence, we can then define a unique average interference perceived by any user in the system when the mean field power strategy is $p(t, X)$, as the limit of the interference term perceived at one receiver side, when the number of users goes to infinity. The resulting interference, which is the response of the mean field, when it implements the power strategy $p(t, X)$ is later referred to as the mean field interference $I(t) = g(p, m)$, and it will be detailed hereafter.

Moreover, our game (4.8) also satisfies the following symmetries requirements, that are necessary in addition to the previously mentioned four hypotheses:

- **Symmetry in control sets:** The players have similar controls p_i and controls sets ($p_i \geq 0$).

- **Symmetry in objective functions:** The players have a common objective function, namely minimizing a twofold objective function consisting of a total power cost function and a penalty function, based on the final packet size remaining.

- **Symmetry in evolution models:** The players states have values in identical sets $\mathcal{H}, \mathcal{Q}$ and similar evolutions models. This is easily verified for the packet size evolutions dQ_i, but it requires that the channel dynamics are the same, at least for the transmission links dh_{ii}. This strong hypothesis is necessary for establishing a symmetry between users and will be discussed in Section 4.10. We also formulate in Proposition 4.5, an approximation for the interfering links.

- **Similar rational behaviors:** The behaviors of users are rational and based on minimizing their respective objective functions.

For a complete and well-written tutorial, we advise the reader to refer to the following paper [161]. The Mean Field Game approximation then simply allows to turn a N-body problem into an equivalent Mean Field Game, which is a 2-body problem only, with:

- **A single AP-UE pair, that we focus on:** this single player can compute the best power strategy $p^*(t, X)$, to use in any state (t, X), in response to any mean field interference $I(t)$. Changing the state (t, X) to compute any optimal strategy $p^*(t, X)$ to use, globally does not affect the outcome of the game, since the number of users is large. From our single user point of view, it is then possible to compute the optimal power strategy $p^*(t, X)$ to be used in any configuration (t, X) in response to a given mean field interference I.

- **The mean field, i.e. the continuum which represents the $N-1$ other pairs:** this mean field generates the mean field interference I, by implementing the unique optimal strategy $p^*(t, X)$ for every user in the continuum.

In the following, we focus on and denote $p(t, X)$ the power strategy to be used by any user in state (t, X) and for any interference pattern I. The objective for the single user consists of finding the optimal power strategy $p^*(t, X)$ to be used by any user in any state (t, X), which is defined according to the following optimization problem:

$$\begin{array}{l} p^*(t, X) = p(t) \text{ in } p = (p(t), ..., p(T)) \\ \text{Where } p = \arg\min_p \left[\mathbb{E}\left[\int_{u=t}^{T} p(u)du + K(Q(T))\right]\right] \\ \text{s.t. } dX(t) = f(t)dt + F(t)dW(t) \\ I(t) \text{ known.} \end{array} \tag{4.23}$$

For any time t and state $X(t) = X$, known. The function $g(p, m)$ refers to how the mean field interference $I(t)$ is processed, when the power strategies used for computing the interference terms are based on $p(t, X)$ and the system evolution is $m(t, X)$: the closed form expression is explicated in the following. Also dX consists of the packet size evolution Q for this user and the direct link channel evolution dh (which replaces dh_{ii}), with similar dynamics as in Equations (4.9) and (4.10):

$$\begin{aligned} Q(t) &= Q(t-1)-\omega(t,X,p)\Delta_t = Q(t-1)-Blog_2\left(1+\frac{p(t)h(t)}{\sigma_n^2+I(t)}\right)\Delta_t \\ dh(t) &= \alpha(t,h(t))dt+\sigma_b dW(t) \end{aligned} \tag{4.24}$$

We now need to define the new interference term $I(t)$with p^* and m, and prove that it converges to a mean field interference when the number of players N goes to infinity. Let us first define the following two empirical PDFs, based on the discrete game:

$$M(t,X) = \frac{1}{N}\sum_{j=1}^{N}\delta_{X_j(t)=X} \xrightarrow{N\to\infty} m(t,X) \tag{4.25}$$

$$M_i'(t,X,h_{int}) = \frac{1}{N}\sum_{j=1}^{N}\delta_{X_j(t)=X}\delta_{h_{ji}(t)=h_{int}} \xrightarrow{N\to\infty} m'(t,X,h_{int}) \tag{4.26}$$

In the previous notations, $M(t,X)$ denotes the proportion of users in state X at time t. When the number of users N goes to infinity, this empirical PDF converge to a mean field density $m(t,X)$, which denotes the state evolution of the players of the system. Similarly, $M_i'(t,X,h_{int})$ denotes the proportion of users in state X at time t, whose interference channels perceived by user i are h_{int}. We also denote $m'(t,X,h_{int})$ the mean field density to which $M_i'(t,X,h_{int})$ converges, which does not depend on i due to the indistinguishability property. $m(t,X)$ and $m'(t,X,h_{int})$ are PDFs, i.e. verify:

$$\int_{h\in\mathcal{H}}\int_{Q\in\mathcal{Q}} m(t,X)dhdQ = 1 \tag{4.27}$$

$$\int_{h_{int}\in\mathcal{H}}\int_{h\in\mathcal{H}}\int_{Q\in\mathcal{Q}} m'(t,X,h_{int})dhdQdh_{int} = 1 \tag{4.28}$$

The original interference term $I_i(t)$ was defined, based on each individual player strategies, as a weighted sum of the other users power strategies:

$$I_i(t) = \frac{1}{N-1}\sum_{\substack{j=1\\ j\neq i}}^{N} h_{ji}(t)p_j(t) \tag{4.29}$$

Using $M(t,X)$ and $M_i'(t,X,h_{int})$, assuming a common power strategy $p^*(t,X)$, and noting $X = (h, Q)$, it rewrites as:

$$I_i(t) = \frac{N}{N-1} \int_{h_{int} \in \mathcal{H}} \int_{h \in \mathcal{H}} \int_{Q \in \mathcal{Q}} M'(t, X, h_{int}) h_{int} p^*(t, X) dh dQ dh_{int} - \frac{1}{N-1} p_i(t) h_{ii}(t) \tag{4.30}$$

The mean field interference term $I(t)$ is then obtained from $I_i(t)$, when N tends to infinity, as:

$$\begin{aligned} I(t) &= \lim_{N \to \infty} I_i(t) \\ &= \lim_{N \to \infty} \frac{N}{N-1} \int_{h_{int} \in \mathcal{H}} \int_{h \in \mathcal{H}} \int_{Q \in \mathcal{Q}} M'(t, X, h_{int}) h_{int} p^*(t, X) dh dQ dh_{int} - \frac{1}{N-1} p_i(t) h_{ii}(t) \end{aligned} \tag{4.31}$$

We have $\lim_{N \to \infty} \frac{N}{N-1} = 1$. Also, assuming that $h_{ii}(t) \in \mathcal{H}$, where $\mathcal{H}$ is bounded and $p_i(t)$ has reasonably low values compared to N (this hypothesis relies on the fact that the power will remain bounded, as the system will attempt to minimize its power cost in the optimization problem (4.23)), we can show that $\lim_{N \to \infty} \frac{1}{N-1} p_i(t) h_{ii}(t) = 0$. The mean field interference term can be approximated as:

$$I(t) \approx \int_{h_{int} \in \mathcal{H}} \int_{h \in \mathcal{H}} \int_{Q \in \mathcal{Q}} M'(t, X, h_{int}) h_{int} p^*(t, X) dh dQ dh_{int} \tag{4.32}$$

We now formulate, in Proposition 4.5, an hypothesis on the interference channels $(h_{ji}(t))_{j \neq i}$ by any user in the system.

Proposition 4.5. *We can define the mean field PDF $\theta(t, h_{int})$, as the PDF limit when N goes to infinity of the following empirical PDF, which is the same for any user i, thanks to the indistinguishability property:*

$$\theta(t, h_{int}) = \frac{1}{N} \sum_{j=1}^{N} \delta_{h_{ji}(t) = h_{int}} \tag{4.33}$$

Basically, we formulate the approximation that the users in the mean field, in any state (t, X) have a probability chance $\theta(t, h_{int})$ of having an interference channel h_{int} affecting the single user we are focusing on. This probability $\theta(t, h_{int})$ does not depend on the users states $X = X(t)$ at time t. The values of $\theta(t, h_{int})$ can be defined using the discrete game, for any single user i, for any time t. Again, $\theta(t, h_{int})$ is a mean field PDF and thus verifies:

$$\int_{h_{int} \in \mathcal{H}} \theta(t, h_{int}) dh_{int} = 1 \tag{4.34}$$

Based on this, the mean field interference term rewrites:

$$I(t) \approx \left(\int_{h_{int}\in\mathcal{H}} \theta(t,h_{int})h_{int}dh_{int}\right)\left(\int_{h\in\mathcal{H}}\int_{Q\in\mathcal{Q}} m(t,X)p^*(t,X)dhdQ\right) \tag{4.35}$$

The interference term decomposes in two terms:

- $\Theta = \int_{h_{int}\in\mathcal{H}} \theta(t,h_{int})h_{int}dh_{int}$ is the average interference channel between the users of the mean field and our single user.

- $\int_{h\in\mathcal{H}}\int_{Q\in\mathcal{Q}} m(t,X)p^*(t,X)dhdQ$ is the average power used for transmission by the users of the mean field.

The interference term was initially defined as the empirical mean of the $N-1$ other users interference terms $(h_{ji}(t)p_j(t))_{j\neq i}$ and its mean field equivalent is then the product of a mean interference channel and a mean power setting. We can then define the final form of the mean field game, equivalent to game (4.8), as:

$$\begin{aligned}
&\forall t, \forall X, p^*(t,X) = p(t) \text{ in } p = (p(t),...,p(T))\\
&\text{Where, } p = \arg\min_p \left[\mathbb{E}\left[\sum_{u=t}^{T} p(u) + K(Q(T))\right]\right]\\
&\text{s.t. } dX(t) = f(t)dt + F(t)dW(t)\\
&I(t) = \Theta\left(\int_{h\in\mathcal{H}}\int_{Q\in\mathcal{Q}} m(t,X)p^*(t,X)dhdQ\right)
\end{aligned} \tag{4.36}$$

With initial state $X(t) = (h(t), Q(t))$, known.

4.5.2 Analysis of the Mean Field Equilibrium

In this section, the objective is to characterize the Mean Field Equilibrium of the Mean Field Game (4.36). Let us first define the Mean Field Equilibrium (MFE) notion, as the equivalent concept to the Nash Equilibrium we defined in Section 4.3 for the N-users stochastic game.

Proposition 4.6. *A power strategy p^* is a Mean Field Equilibrium, if there does not exist a state (t', X') and power control value $p_m \geq 0$, such that:*

$$\forall t, \forall X, C(t,X,p^*) > C(t,X,p') \tag{4.37}$$

Where p' is the power strategy defined as follows:

$$\forall t, \forall X, p'(t,X) = \begin{cases} p_m & \textit{if } (t,X) = (t',X') \\ p^*(t,X) & \textit{else} \end{cases} \tag{4.38}$$

Also and as in Proposition 4.2, the Mean Field Game (4.36) is well-defined: under general conditions, we can show that there exist a unique MFE [63, 162].

In order to define the equations that rule the Mean Field Equilibrium, we proceed as in Section 4.3. In the end, the Mean Field Equilibrium can be defined with a set of two fundamental equations, namely:

- The Hamilton-Jacobi-Bellman (HJB) equation (4.42): it models the best power strategy $p(t,X)$ to be used in any state (t,X), in response to a given mean field interference $I(u), u \in [t,T]$.
- The Fokker-Planck-Kolmogorov (FPK) equation (4.45): it models the evolution of all the users in the system, through the density of users in state (t,X), that we denoted $m(t,X)$ in the previous section. The evolution of m depends on the given power strategy $p(t,X)$, that the users implement. When the evolution of the density m is known, we can define the mean field interference I, as the response of the mean field to the power strategy $p(t,X)$.

In that sense, we observe that the Mean Field Game is a 2-body problem with a set of two backward-forward equations, modeling the behavior of each body respectively. We summarize the Mean Field Game in Figure 4.4.

Let us now focus on defining the best power strategy to be used in response to a mean field interference $I(u), u \in [t,T]$. We first define the running cost function $v(t,X,p)$, as:

$$v(t,X,p) = \mathbb{E}\left[\sum_{u=t}^{T} p(u)du + K(Q(T)\right] \tag{4.39}$$

Where $Q(T) = Q(t) - \sum_{u=t}^{T} B\log_2\left(1+\frac{p(t,X(t))h(t)}{\sigma_n^2+I(t)}\right)$ and $X(t) = (Q(t),h(t))$.

Similarly to the N-users stochastic game, the Mean Field Equilibrium p^* can be defined according to $v^*(t,X,p^*)$, the trajectory of the running cost functions when $p = p^*$. More specifically, v^* verifies the Hamilton Jacobi Bellman (HJB) equation, a backward PDE detailed in Proposition 4.7:

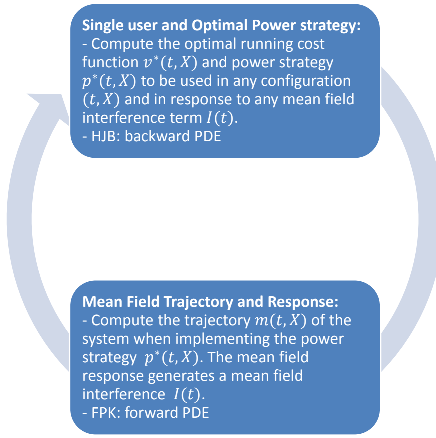

Figure 4.4: Summary of the Mean Field Game.

Proposition 4.7. *The optimal running cost trajectory $v^*(t,X)$ is the unique solution to the HJB equation [63]:*

$$\forall t, \forall X, \min_{p(t,X)} \left[p(t,X)+\alpha(t,h)\partial_h v^*(t,X)-\omega(t,X,p)\partial_Q v^*(t,X)+\frac{1}{2}\sigma_b^2\partial_{hh}v^*(t,X)\right]+\partial_t v^*(t,X)=0 \tag{4.40}$$

With the final condition $v^*(T,X(T)) = K(Q(T))$ Also, $p^*(t,X)$ is defined as the solution of the infimum term, i.e. $\forall t, \forall X$:

$$\begin{aligned} p^*(t,X) &= \arg\min_{p(t,X)} \left[p(t,X)+\alpha(t,h)\partial_h v^*(t,X)-\omega(t,X,p)\partial_Q v^*(t,X)+\frac{1}{2}\sigma_b^2\partial hhv^*(t,X)\right] \\ &= \left(\frac{B}{log(2)}\partial_Q v^*(t,X)-\frac{\sigma_n^2+I(t)}{h}\right)^+ = \max\left(0, \frac{B}{log(2)}\partial_Q v^*(t,X)-\frac{\sigma_n^2+I(t)}{h}\right) \end{aligned} \tag{4.41}$$

We can observe that the optimal power strategy resembles to a time water-filling solution, as it was described in Section 4.3.1, by posing μ the water level equal to $\frac{B}{log(2)}\partial_Q v^*(t,X)$. Reusing this expression of $p^*(t,X)$ in the infimum term leads to the final version of the HJB equation:

$$\begin{aligned} \forall t, \forall X, \quad & \frac{\sigma_n^2+I(t)}{h}+\alpha(t,h)\partial_h v^*(t,X)-\left[\frac{B}{log(2)}-B\log_2\left(\frac{Bh\partial_Q v^*(t,X)}{log(2)(\sigma_n^2+I(t))}\right)\right]\partial_Q v^*(t,X) \\ & +\frac{1}{2}\sigma_b^2\partial_{hh}v^*(t,X)+\partial_t v^*(t,X)=0 \end{aligned} \tag{4.42}$$

With the final condition $v^*(T,X(T)) = K(Q(T))$

We have then defined the first fundamental equation for the Mean Field

Game, the HJB equation (4.42). The optimal running cost function $v^*(t, X)$ is obtained via a backward PDE: the optimal running cost function is then chosen by backward reasoning, which is standard in optimal control theory, and differential game theory. We observed it as well in Section 3.3. The second equation, namely the FPK equation, is a forward PDE, which models the forward evolution of the system $m(t, X)$, when the users implement the mean field power strategy $p(t, X)$. It is defined in Proposition 4.8.

Proposition 4.8. *When the power strategy is $p(t, X)$, the evolution of the system density $m(t, X)$ is then the unique solution to the FPK equations, which is defined as the unique solution to the following PDE:*

$$\partial_t m(t, X) = -\partial_h \left[\alpha(t, h) m(t, X)\right] - \partial_Q \left[\omega(t, X, p) m(t, X)\right] - \frac{1}{2}\sigma_b^2 \partial_{hh} m(t, X) \tag{4.43}$$

With the initial users states density, $m_0(X) = m(0, X)$, known, as it is based on the discrete game, as:

$$m_0(X) = \frac{1}{N} \sum_{i=1}^{N} \delta_{X_i(0)=X} \tag{4.44}$$

Reusing the expression of p^* from equation (4.41), we obtain the final form for the FPK equation:

$$\begin{aligned} &\partial_t m(t, X) + \alpha(t, h)\partial_h m(t, X) + m(t, X)\partial_h \alpha(t, h) - \omega(t, X, p)\partial_Q m(t, X) \\ &- m(t, X)\partial_Q \omega(t, X, p) + \frac{1}{2}\sigma_b^2 \partial_{hh} m(t, X) = 0 \end{aligned} \tag{4.45}$$

With the initial users states density, $m_0(X) = m(0, X)$, known. Also,

$$\partial_Q \omega(t, X, p) = \frac{B}{log(2)} \frac{\partial_{QQ} v^*(t, X)}{\partial_Q v^*(t, X)} \tag{4.46}$$

We have then defined the two fundamental PDEs used for defining the Mean Field Equilibrium, namely the HJB (4.42) and the FPK (4.45). The solution of the two coupled forward-backward PDEs in (v^*, m) gives the optimal trajectory, and can then be used in order to compute both:

- the mean field interference $I(t)$, according to Equation (4.35),
- and the optimal mean field power strategy $p^*(t, X)$, according to Equation (4.41).

In the next section, we will focus on defining the optimal trajectory (v^*, m) which solves the two fundamental coupled PDEs.

4.5.3 An Iterative Method for Approaching the Mean Field Equilibrium

We have shown in the previous section, that we could compute the optimal mean field power strategy p^*, if we could find the solution couple (v^*, m) to the system of two coupled PDEs. In order to approach the solution couple (v^*, m), we consider the following iterative process, inspired from [163, 161]. In the following, we denote $Y_{(i)}$, the value of the parameter $Y = \{p, m, v, I, ...\}$ at iteration i.

At the beginning of the algorithm, the interference term $I_{(0)}$, power strategies $p^*_{(0)}$, running cost functions $v^*_{(0)}$ and users evolution density $m_{(0)}$ are initialized with zeros values. Then, repeat alternatively the following steps, until a convergence is (hopefully) observed on (v^*, m).

- Increase the iteration number i by one.

- Solve the HJB equation for $v^*_{(i)}$, assuming the interference term is fixed to $I_{(i-1)}$, which is defined $\forall t, \forall X$, as:

$$\frac{\sigma_n^2 + I_{(i-1)}(t)}{h} + \alpha(t,h)\partial_h v^*_{(i)}(t,X) - \left[\frac{B}{log(2)} - B\log_2\left(\frac{Bh\partial_Q v^*_{(i-1)}(t,X)}{log(\sigma_n^2 + I_{(i-1)})(t)}\right)\right]\partial_Q v^*_{(i)}(t,X)$$
$$+\frac{1}{2}\sigma_b^2\partial_{hh}v^*_{(i)}(t,X) + \partial_t v^*_{(i)}(t,X) = 0 \tag{4.47}$$

  With the final condition $v^*_{(i)}(T, X(T)) = K(Q(T))$.

- updates the power strategy $p_{(i)}$, with the new value of $v^*_{(i)}$ obtained from the previous HJB and the mean field interference term $I_{(i)}$, as:

$$p^*_{(i)}(t,X) = \frac{B}{log(2)}\partial_Q v^*_{(i)}(t,X) - \frac{\sigma_n^2 + I_{(i-1)}(t)}{h} \tag{4.48}$$

$$I_{(i)}(t) = \Theta\left(\int_{h\in\mathcal{H}}\int_{Q\in\mathcal{Q}} m_{(i-1)}(t,X)p^*_{(i)}(t,X)dhdQ\right) \tag{4.49}$$

- Solve the FPK equation for $m_{(i)}$, assuming the interference term $I = I_{(i)}$

and $v^* = v^*_{(i)}$ are fixed:

$$\partial_t m_{(i)}(t,X) + \alpha(t,h)\partial_h m_{(i)}(t,X) + m_{(i)}(t,X)\partial_h \alpha(t,h) - \omega(t,X,p)\partial_Q m_{(i)}(t,X) \\ - m_{(i)}(t,X)\partial_Q \omega(t,X,p) + \frac{1}{2}\sigma_b^2 \partial_{hh} m_{(i)}(t,X) = 0 \tag{4.50}$$

With the initial users states density, $m_0(X) = m(0,X)$, known. And $\partial_Q \omega(t,X,p)$, is defined with $v^* = v^*_{(i)}$, as:

$$\partial_Q \omega(t,X,p) = \frac{B}{log(2)} \frac{\partial_{QQ} v^*_{(i)}(t,X)}{\partial_Q v^*_{(i)}(t,X)} \tag{4.51}$$

- Re-update the mean field interference term $I_{(i)}$ and power strategy $p_{(i)}$, with the new values of $m_{(i)}$ obtained from the previous equation, as:

$$p^*_{(i)}(t,X) = \frac{B}{log(2)} \partial_Q v^*_{(i)}(t,X) - \frac{\sigma_n^2 + I_{(i)}(t)}{h} \tag{4.52}$$

$$I_{(i)}(t) = \Theta\left(\int_{h\in\mathcal{H}} \int_{Q\in\mathcal{Q}} m_{(i)}(t,X) p^*_{(i)}(t,X) dhdQ \right) \tag{4.53}$$

- Compute a convergence criterion on $p^*_{(i)}$, for example if there has been a limited variation on any element of p^* between the previous iteration $(i-1)$ and the present iteration (i), which can be formulated as follows:

$$\max_{t,X} \left[|p^*_{(i-1)}(t,X) - p^*_{(i)}(t,X)| \right] \gtrless \epsilon_{cv} \tag{4.54}$$

With ϵ_{cv} known and relatively small, depending on the precision wanted.

The process is known to converge to a fixed point, which consists of the Mean Field Equilibrium [163, 161]. When a convergence is observed on $p^*_{(i)}$, the values obtained on the last iteration, approach the Mean Field Equilibrium. The process is quite similar to the one detailed in Algorithm 1, where the algorithm alternatively adapted the power strategy of a random user assuming the interference term was fixed and then updated the interference perceived by the other users of the system. In the Mean Field Game, the interference term is perceived by every user in the system and is defined according to the mean field power strategy exclusively, which allows for a facilitated convergence for the iterative algorithm, described hereafter. The process was then repeated until a convergence was observed. This iterative process is known to be of slow convergence, as it depends on the number of independent users in the system N. However, by transitioning to a Mean Field Game, we simplified the problem, as it became a 2-body problem only, for any number of users N, supposed large. By analogy, we could observe that the proposed iterative process has a computational complexity equivalent to those of the algorithm 1 when $N = 2$, as there is only two bodies in the Mean Field Game: one user of interest and the mean field. Transitioning into a mean field game presents a great advantage, as it allows to reduce drastically the computation complexity of any system of N (N supposed large) independent users, from N to only 2, which is a notably more tractable scenario. Moreover, the mean field control p^* is a good approximation to the optimal power strategy to be used by any discrete user of the N-users stochastic game.

$$\forall i \in \mathcal{N}, \forall t, \forall X = X(t), p_i^*(t) \approx p^*(t, X) \tag{4.55}$$

In [62], the authors have also shown that the approximation quality increases when the number of independent players N goes to infinity.

Moreover, we can observe that fixing the interference term I, power strategy p^*, and m in the HJB equations allowed to linearize the HJB equation, turning it into a standard form PDE:

$$\begin{aligned}&d(t,X)\partial_t v^*(t,X) + a_h(t,X)\partial_h v^*(t,X) + a_Q(t,X)\partial_Q v^*(t,X) + a_{hh}(t,X)\partial_{hh} v^*(t,X)\\&+b(t,X)v^*(t,X) + c(t,X) = 0\end{aligned} \tag{4.56}$$

The coefficients $a_., b, c, d$ in front of every derivative are fixed expression wrt. (t, X), based on the previous iterations of I, p^*, m and v^*. The same way, the FPK can also be 'linearized' into an equation of the same standard form. This linearization, also allowed to simplify the computation of the solutions to each PDE. The method used for numerically approximating the two PDEs at each iteration is based on finite differences and is strongly inspired from [164, 165, 166]. In the next sections, we analyze the performance of the 3 detailed power strategies in 4 channel models scenarios, with increasing complexity.

4.6 Channel Model 1: Constant and Equal Channels

4.6.1 Introduction and Optimization

In this first section, we assume that the channels are non time-varying, constant and equal to one, i.e. $\forall i, j, t, h_{ij}(t) = 1$. This assumption has been discussed in other works on power control [61, 167, 153, 67], in particular, it is relevant in scenarios in which the channels are subject to fast fading or interpreted as a limiting case for slow fading channels. Moreover, this first scenario allows for great simplifications in the MFG PDEs, as the optimal power strategies will not take into account the variations of the channels. As a consequence, the analysis of the Mean Field strategies is simplified, but still provides good insights on how the Mean Field Strategies work, as well as their performance compared to the two reference strategies, detailed in Section 4.4. Our objective in this section consists of finding the set of optimal power strategies p^*, as the set of the optimal power strategies of each user i, denoted p_i^*, solution to the simplified version of the optimization problem (4.8), defined hereafter:

$$\begin{aligned} &p_i^* = (p_i^*(1), ..., p_i^*(T)) = \arg\min_{p_i} \left[\mathbb{E}\left[\textstyle\sum_{t=1}^{T} p_i(t)\right]\right] \\ &\text{s.t. } Q_i(t) = Q_i(t-1) - Blog_2\left(1+\gamma_i(t)\right)\Delta_t \\ &Q_i(T) = 0; \end{aligned} \tag{4.57}$$

Where

$$\gamma_i(t) = \frac{p_i(t)}{\sigma_n^2 + \frac{1}{N-1}\sum_{\substack{j=1 \\ j\neq i}}^{N} p_j(t)} \tag{4.58}$$

4.6.2 Optimal Strategies with Time Water-Filling

In this section, we demonstrate in this section that the set of optimal power strategies $p^* = (p_1^*, ..., p_N^*)$ can be analytically computed in a simple scenario where the channels are constant wrt time. In that sense, we investigate an approach for finding the Nash Equilibrium related to game (4.57). Let us first recall, that we demonstrated in Section 4.3, that the power strategies of the Nash Equilibrium are necessarily time water-filling strategies. Since there are no time variations of the channels, we can also observe, as suggested in [168], that the optimal power strategies p_i^*, and interference terms I_i^* are going to be constant wrt time. In such a constant channels scenario, the unique optimal power strategies set $P = (p_1^*, ..., p_N^*)'$ can be obtained as the unique solution to the set of N linear equations defined in Equation (4.59):

$$\begin{aligned} &\forall i \in \mathcal{N}, Q_i(T) = Q_i(0) - \sum_{t=1}^{T} B log_2(1 + \frac{p_i^*}{\sigma_n^2 + \sum_{\substack{j=1 \\ j \neq i}}^{N} p_j^*}) = 0 \\ &\Leftrightarrow \forall i \in \mathcal{N}, \sigma_n^2 \left(2^{\frac{Q_i(0)}{BT\Delta_t}} - 1\right) = p_i^* + \frac{1}{N-1}\left(1 - 2^{\frac{Q_i(0)}{BT\Delta_t}}\right) \sum_{\substack{j=1 \\ j \neq i}}^{N} p_j^* \end{aligned} \tag{4.59}$$

In order to solve the set of N linear equations $AP' = B$ (where P' is the transposed vector P, i.e. denotes the vector P in column notation), one must invert a matrix A, of size $N \times N$, whose general term A_{ij} is then defined as:

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \\ \frac{1}{N-1}\left(1 - 2^{\frac{Q_i(0)}{BT\Delta_t}}\right) & \text{else} \end{cases} \tag{4.60}$$

And B is the N elements vector, with general term $B_i = -\left(1 - 2^{\frac{Q_i(0)}{BT\Delta_t}}\right)$.

Also, we can observe that since the channels, powers and interference terms are constant with respect to time, the packet evolution $Q_i(t)$ of any user in the system will uniformly decrease at a constant rate. As a consequence, it appears, that when the channel is constant wrt time, the optimal strategy consists, in fact, of an equal-bit strategy, similar to the one we described in a previous chapter (Section 3.4.3). And, in such a strategy, the powers to be used at the beginning of each channel can be computed without any knowledge of the future channel realizations. This proves, again, that accessing a future knowledge in a delay-tolerant network does not lead to a performance gain compared to heuristic strategies such as the equal-bit strategy, in scenarios where there are no time variations of the channels.

4.6.3 Updated MFG PDEs

Since the channels are not time-varying, the optimal power strategies are necessarily constant wrt time: a first possibility for computing the optimal strategies p^* is then given by the *Constant Power* strategy. However, it requires to invert a matrix of size $N \times N$, which can rapidly become time consuming, when the number of users N in the system becomes large. A second possibility consists of transitioning into a Mean Field Game, with reduced complexity, since it only consists of 2 bodies, compute the optimal mean field power strategy, and use it for computing the individual power strategies of each user in the system. Our procedure transitions from any number of users N to an equivalent game with reduced complexity. As a consequence, the Mean Field approach has a constant computational cost, whatever the initial number of users N was. As mentioned before, assuming constant channels with identical values allows for great simplifications in the MFG PDEs, as the optimal power strategies will not take into account the variations of the channels. More specifically, we can first remove all the partial derivatives wrt h in both the HJB and FPK equations, as we have $\alpha_{ij}(t, h_{ij}) = 0$, $\alpha(t, h) = 0$ and $\sigma_b^2 = 0$. Also, we have $\Theta = 1$.

The optimal running cost trajectory v^* and power p^* are now functions of t and Q only.

$$\begin{aligned} p^*(t,Q) &= \arg\min_{p(t,Q)} \left[p(t,Q) - \omega(t,Q,p)\partial_Q v^*(t,Q)\right] \\ &= \tfrac{B}{log(2)}\partial_Q v^*(t,Q) - (\sigma_n^2 + I(t)) \end{aligned} \tag{4.61}$$

And the HJB equation is then simplified as:

$$\forall t, \forall Q, (\sigma_n^2 + I(t)) - \left[\frac{B}{log(2)} - B\log_2\left(\frac{B\partial_Q v^*(t,Q)}{log(2)(\sigma_n^2 + I(t))}\right)\right]\partial_Q v^*(t,Q) + \partial_t v^*(t,Q) = 0 \tag{4.62}$$

With the final condition $v^*(T, Q) = K(Q(T))$.

And the FPK equation is simplified as:

$$\partial_t m(t,Q) - B\log_2\left(\frac{B\partial_Q v^*(t,Q)}{log(2)(\sigma_n^2 + I(t))}\right)\partial_Q m(t,Q) - m(t,Q)\partial_Q \omega(t,Q,p) = 0 \tag{4.63}$$

With the initial distribution of packets to transmit, $m_0(Q) = m(0, Q)$, known. Also, $\partial_Q \omega(t, Q, p) = \frac{B}{log(2)} \frac{\partial_{QQ} v^*(t,Q)}{\partial_Q v^*(t,Q)}$.

Finally, the interference term can be simply computed as:

$$I(t) = \int_{Q\in\mathcal{Q}} m_{(i)}(t,Q)p^*(t,Q)dQ \tag{4.64}$$

4.6.4 Simulation Results

4.6.4.1 Simulation Parameters and Performance Criterion

In this section, we investigate the performance of the three previously detailed strategies, namely:

- The *Full-Power strategy*, detailed in Section 4.4.2,

- The *Constant Power strategy*, detailed in Section 4.6.2,
- The *Mean Field strategy*, obtained by first computing the Mean Field Equilibrium, following the equations from Section 4.6.3, and then applying the mean field power strategy to the N users, in order to obtain the power strategy for each individual user, as suggested in Equation (4.55).

The performance criterion to be considered for each strategy under investigation, is the average cumulated power cost per user E, which depends on the power strategy p_i^* of each user i:

$$E = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} p_i^*(t) \tag{4.65}$$

In order to observe the average performance of each strategy under investigation in terms of cumulated power cost per user, we run Monte-Carlo simulations with $N_{MC} = 1000$ independent iterations. The parameters used for simulation are listed in Table 4.1.

Parameter	Parameter Value
Number of users N	1000
Number of time slots T	20
Time slot duration Δ_t and Bandwidth B	$B\Delta_t = 0.001$
Noise variance σ_n^2	1
Maximal packet size Q_{max}	100
Resolution for packet size set	50 elements
Initial packet sizes $Q_i(0)$	Uniformly distributed over the 50 elements of the packet size set

Table 4.1: Simulation parameters

We must also define the penalty function $K(Q(T))$ to be used in our simulations. As mentioned in Section 4.3, it must be a continuous function, with no penalty when the transmission is complete ($Q(T) = 0$), but should strongly penalized a non complete transmission ($Q(T) > 0$). A simple function that immediately comes to mind is the Heaviside function. However, the Heaviside function is not smooth, for this reason, we prefer the logistic function, initially studied by Pierre FranÃ§ois Verhulst [169]. We have represented in Figure 4.5, the logistic function, based on Equation (4.66): it returns no penalty when $Q(T) = 0$ and a penalty $K(Q(T) \approx \frac{\phi}{2}$, when $Q(T) > 0$. In our numerical simulations to follow, we consider $\phi = 10000$ and $\rho = 100$, where ϕ, and ρ are constants representing the maximum value and the steepness of the curve respectively.

$$K(Q(T)) = \frac{\phi}{1+e^{-\rho Q(T)}} - \frac{\phi}{2} \tag{4.66}$$

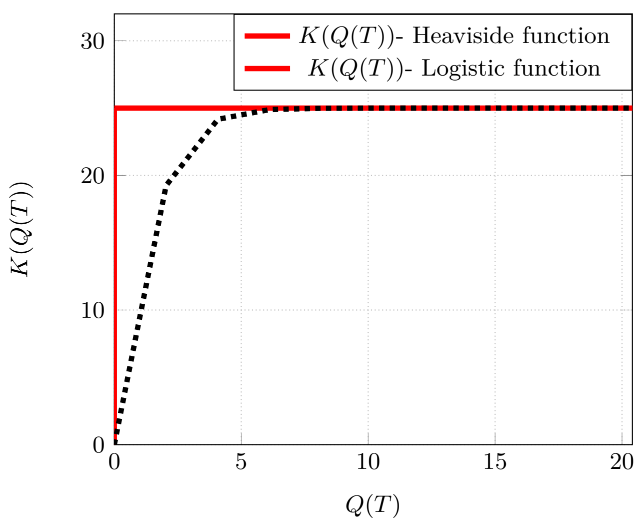

Figure 4.5: Standard logistic sigmoid function, with $\phi = 50$ and $\rho = 1$

4.6.5 Analysis of the Mean Field Equilibrium and the Mean Field Strategy

In this section, we present the graphs related to the Mean Field evolution $m(t, Q)$, as well as the graph representing the Mean Field power strategy $p^*(t, Q)$, for a single Monte-Carlo realization. Let us first focus on the evolution of the population represented in Figure 4.6. At t = 0, we observe an initial uniform distribution of users over the set of packet sizes $Q(0)$. At $t = T$, almost every user has achieved its transmission as requested, only a little proportion of users have a very small remaining packet to receive, therefore accepting the penalty function. This suggests an appropriate choice of the penalty function. In Figure 4.7, we represented the Mean Field power strategy $p^*(t, Q)$. The instantaneous power strategy $p^*(t, Q)$ to be used at time t assuming $Q(t) = Q$ increases when the remaining packet size increases, but also increases when the system gets closer to the deadline.

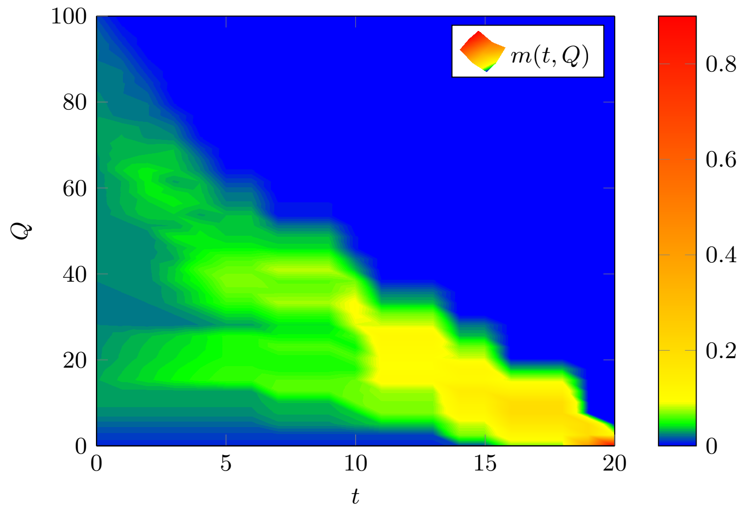

Figure 4.6: Optimal distribution of users $m(t, Q)$ whose packet size at time slot t (x-axis) is Q (y-axis). Channel model 1

4.6.6 Performance Analysis of the Investigated Strategies

In this section, we investigate the behavior of the three strategies under investigation for a single Monte-Carlo realization. In the following we investigate the behavior of three users with different initial packet sizes $Q(0) = 20, 50, 100$, and study the performance of the three power strategies under investigation. In Figure 4.8, we represent the instantaneous power strategy used by every single user in each strategy under investigation. It appears that the Mean Field Strategy and the Constant power strategy are closely related. This means that the *Mean Field* strategy is able to approach notably the optimal power strategy, which is given by the *Constant Power* strategy, as detailed in Section 4.6.2. As expected, the *Full Power* strategy transmits at full power, in order to complete the transmission as soon as possible. In Figure 4.9, we present the packet size evolution for our 3 users, and observe that the optimal strategy transmits the same amount of packet on every single time slot, and so does the *Mean Field* strategy. Both strategies are then strictly equivalent to an equal-bit scheduler, as previously discussed in Section 4.6.2. The *Full Power* strategy on the other

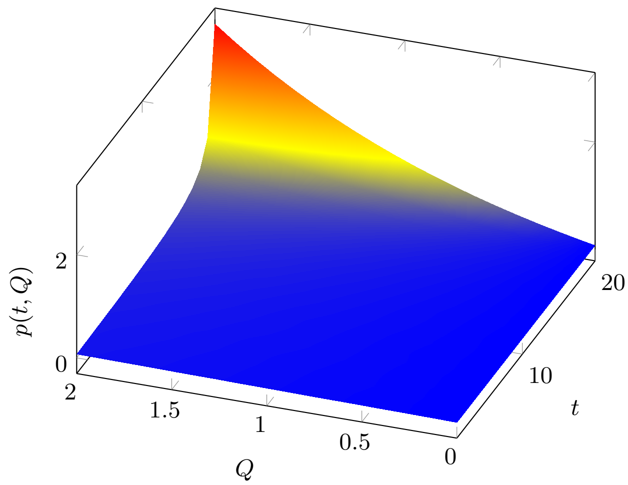

Figure 4.7: Optimal instantanous Mean Field power strategy $p(t, Q)$ to be used by any user whose packet size at time slot t (x-axis) is Q (y-axis). Channel model 1.

hand is unable to schedule its transmission over the latency of T time slots offered to the system. We can observe on Figure 4.10 the cumulated power costs for each user and each strategy. We observe that the *Mean Field* strategy has a cumulated power cost close to the optimal *Constant Power* strategy, whereas the *Full Power* strategy has a poor energy efficiency. This demonstrates that the Mean Field is a good approximation of the optimal strategy to be used by any user in the system. This also highlights a significant performance gain, compared to *Full Power* strategies, which is simply due to the energy efficiency vs. latency trade-off. In order to observe the significance of the potential gain offered by an optimal strategy compared to a state-of-the-art *Full Power* strategy, we represented on Figure 4.11, the histogram of average Energy Cumulated Cost per user, over the N_{MC} Monte-Carlo realizations, for each strategy under investigation. Again, we observe that the *Mean Field* and the *Constant Power* strategies have close performance. We also observe a significant gain compared to a *Full Power* strategy. On average, the *Mean Field* and the *Constant Power* strategy allows for an average reduction of the cumulated energy cost per user of 44% compared to the *Full Power* strategy. The presented simulations highlight the significance of the potential performance gain that can be obtained by exploiting the energy efficiency vs. latency trade-off, and also shows that the *Mean Field* strategy closely approaches the optimal performance. However, there is no gain, in knowing the future, since the optimal power strategy could have been computed using an unaware scheduler, namely the equal-bit scheduler. There will be a gain, obtained by exploiting future knowledge, when the channel becomes time varying, which is detailed in Section 4.8. In the next section, we assume that the channels remain constant over time, but they may now take different values in $\mathcal{H}$.

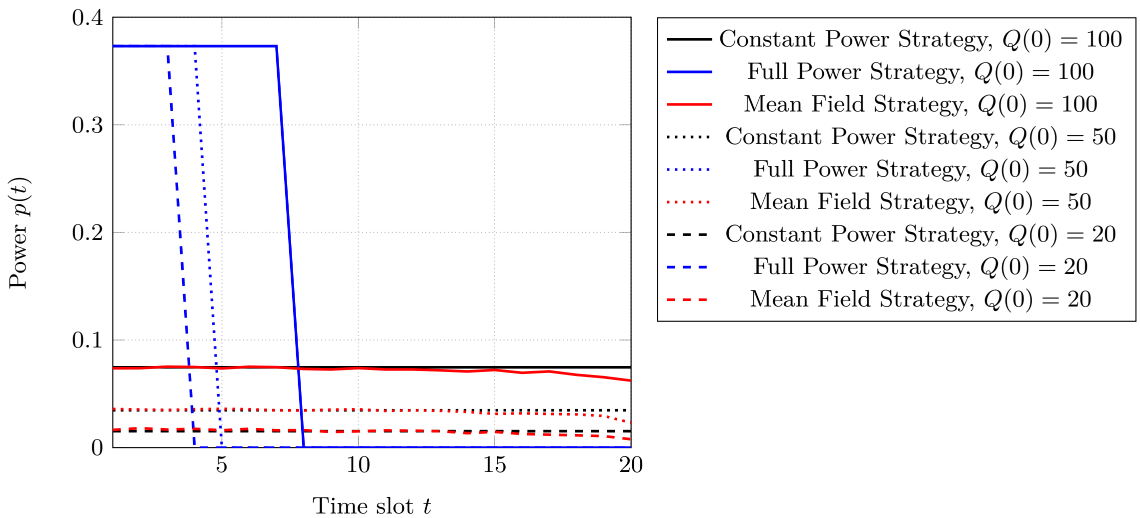

Figure 4.8: Instantaneous power strategies $p(t)$ for 3 strategies, 3 different initial packet sizes $Q(0) = 20, 50, 100$. Channel model 1.

4.7 Channel Model 2: Constant Channels

4.7.1 Introduction and Optimization

In this section, we assume that the channel are remain constant wrt time, but can take different values, i.e. $\forall i, j, t, h_{ij}(t) = \bar{h}_{ij}$. This channel model is an improvement of the previous constant and equal channel model, as the system now has to adapt the instantaneous power strategies to an additional component, i.e. the channels $\bar{h}_{ij}$. Although more complex, it still allows for simplifications, as the channel still do not evolve wrt time. As a consequence, the analysis of the Mean Field strategies is simplified, but still provides good insights on how the Mean Field Strategies work, as well as their performance compared to the two

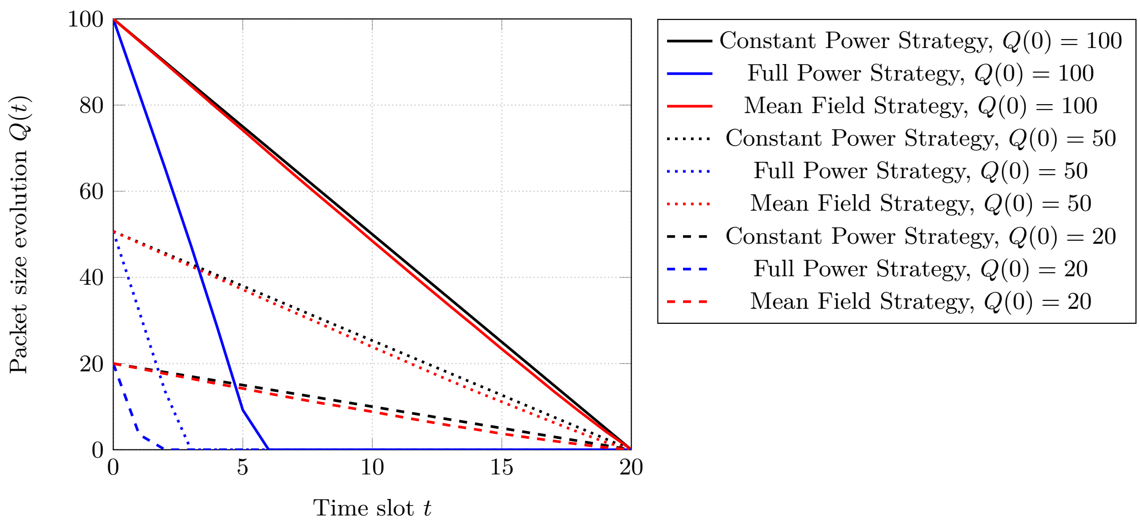

Figure 4.9: Packet Sizes Evolutions $Q(t)$ for 3 strategies, 3 different initial packet sizes $Q(0) = 20, 50, 100$. Channel model 1.

reference strategies, detailed in Section 4.4. Our objective in this section consists of finding the set of optimal power strategies p^*, as the set of the optimal power strategies of each user i, denoted p_i^*, solution to the simplified version of the optimization problem (4.8), defined hereafter:

$$\begin{aligned} & p_i^* = (p_i^*(1), ..., p_i^*(T)) = \arg\min_{p_i} \left[\mathbb{E} \left[\textstyle\sum_{t=1}^{T} p_i(t) \right] \right] \\ & \text{s.t. } Q_i(t) = Q_i(t-1) - Blog_2\left(1+\gamma_i(t)\right) \Delta_t \\ & Q_i(T) = 0; \end{aligned} \tag{4.67}$$

Where

$$\gamma_i(t) = \frac{\bar{h}_{ii} p_i(t)}{\sigma_n^2 + \frac{1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{h}_{ji} p_j(t)} \tag{4.68}$$

4.7.2 Optimal Strategies with Time Water-Filling

In this section, and as in Section 4.6.2, we demonstrate in this section that the set of optimal power strategies $p^* = (p_1^*, ..., p_N^*)$ can be analytically computed in a simple scenario where the channels are constant wrt time. In that sense, we investigate an approach for finding the Nash Equilibrium related to game (4.67). Again and for the same reasons we pointed out precedently, the power

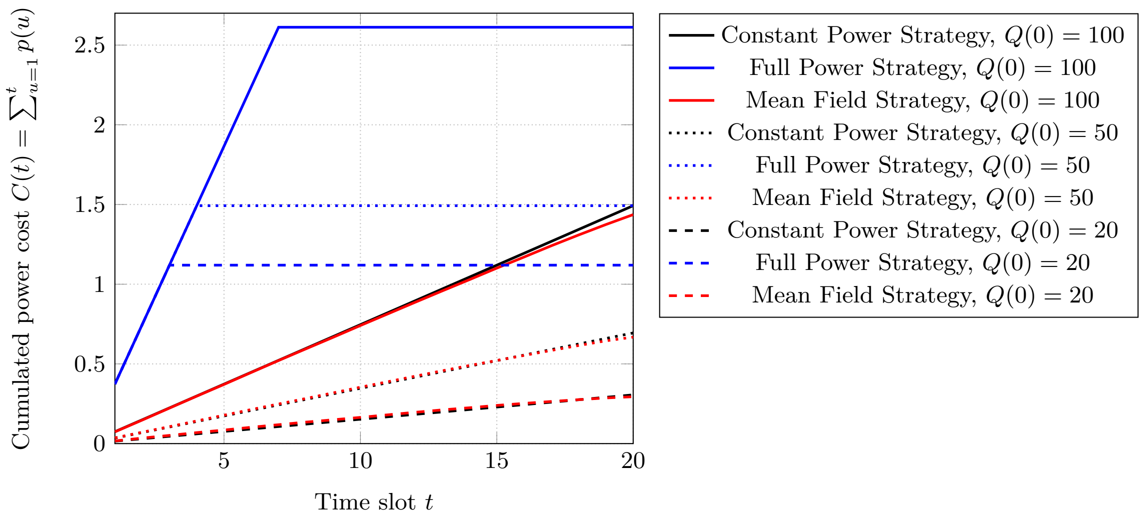

Figure 4.10: Cumulated power cost $C(t) = \sum_{u=1}^{t} p(u)$ for 3 strategies, 3 different initial packet sizes $Q(0) = 20, 50, 100$. Channel model 1.

strategies of the Nash Equilibrium are necessarily time water-filling strategies. Furthermore, the optimal power strategies p_i^*, and interference terms I_i^* are going to be constant wrt time t. In such a constant channels scenario, the unique optimal power strategies set $P = (p_1^*, ..., p_N^*)'$ can be obtained as the unique solution to the set of N linear equations defined in Equation (4.69):

$$\begin{aligned} &\forall i \in \mathcal{N}, Q_i(T) = Q_i(0) - \sum_{t=1}^{T} Blog_2\Big(1 + \frac{\bar{h}_{ii} p_i^*}{\sigma_n^2 + \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{h}_{ji} p_j^*}\Big) = 0 \\ &\Leftrightarrow \forall i \in \mathcal{N}, \sigma_n^2 \left(2^{\frac{Q_i(0)}{BT\Delta_t}} - 1\right) = \bar{h}_{ii} p_i^* + \frac{1}{N-1}\left(1 - 2^{\frac{Q_i(0)}{BT\Delta_t}}\right) \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{h}_{ji} p_j^* \end{aligned} \quad (4.69)$$

In order to solve the set of N linear equations $AP' = B$, one must invert a matrix A, of size $N \times N$, whose general term A_{ij} is then defined as:

$$A_{ij} = \begin{cases} \bar{h}_{ii} & \text{if } i = j \\ \frac{1}{N-1} \bar{h}_{ji} \left(1 - 2^{\frac{Q_i(0)}{BT\Delta_t}}\right) & \text{else} \end{cases} \quad (4.70)$$

And B is the N elements vector, with general term $B_i = -\left(1 - 2^{\frac{Q_i(0)}{BT\Delta_t}}\right)$.

Also, we can observe that since the channels, powers and interference terms

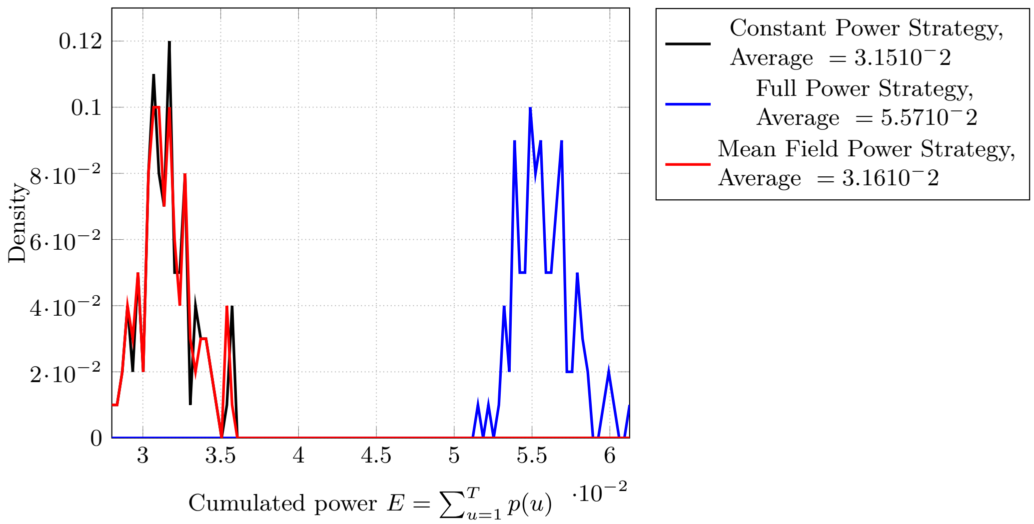

Figure 4.11: Histogram of the final cumulated power cost $E = \sum_{t=1}^{T} p(t)$ over the N_{MC} independent Monte-Carlo Realizations, for the 3 strategies. Channel model 1.

are constant with respect to time, the packet evolution $Q_i(t)$ of any user in the system will uniformly decrease at a constant rate. As a consequence, it appears, that when the channel is constant wrt time, the optimal strategy consists, in fact, of an equal-bit strategy, similar to the one we described in a previous chapter (Section 3.4.3). And, in such a strategy, the powers to be used at the beginning of each channel can be computed without any knowledge of the future channel realizations. This proves, again, that accessing a future knowledge in a delay-tolerant network does not lead to a performance gain compared to heuristic strategies such as the equal-bit strategy, in scenarios where there are no time variations of the channels.

4.7.3 Updated MFG PDEs

Since the channels are not time-varying, the optimal power strategies are necessarily constant wrt time: a first possibility for computing the optimal strategies p^* is then given by the *Constant Power* strategy. However, it requires to invert a matrix of size $N \times N$, which can rapidly become time consuming, when the number of users N in the system becomes large. A second possibility consists of transitioning into a Mean Field Game, with reduced complexity, since it only consists of 2 bodies, compute the optimal mean field power strategy, and use it for computing the individual power strategies of each user in the system. Our procedure transitions from any number of users N to an equivalent game with reduced complexity. As a consequence, the Mean Field approach has a constant computational cost, whatever the initial number of users N was. As mentioned before, assuming constant channels with identical values allows for great simplifications in the MFG PDEs, as the optimal power strategies will not take into account the variations of the channels. More specifically, we can first remove all the partial derivatives wrt h in both the HJB and FPK equations, as we still have $\alpha_{ij}(t, h_{ij}) = 0$, $\alpha(t, h) = 0$ and $\sigma_b^2 = 0$. But, the channel h now appears among the state variable to be considered in the MFG PDEs.

The optimal running cost trajectory v^* and power p^* are now functions of t, $X = (h, Q)$.

$$\begin{aligned} p^*(t,X) \quad &= \arg\min_{p(t,X)} \left[p(t,X) + \alpha(t,h)\partial_h v^*(t,X) - \omega(t,X,p)\partial_Q v^*(t,X) + \frac{1}{2}\sigma_b^2 \partial hhv^*(t,X) \right] \\ &= \tfrac{B}{log(2)}\partial_Q v^*(t,X) - \tfrac{\sigma_n^2 + I(t)}{h} \end{aligned} \tag{4.71}$$

The HJB equation can be simplified as:

$$\forall t, \forall X, \frac{\sigma_n^2 + I(t)}{h} - \left[\frac{B}{log(2)} - B\log_2\left(\frac{Bh\partial_Q v^*(t,X)}{log(2)(\sigma_n^2 + I(t))} \right) \right] \partial_Q v^*(t,X) + \partial_t v^*(t,X) = 0 \tag{4.72}$$

With the final condition $v^*(T, X(T)) = K(Q(T))$

We have then defined the first fundamental equation for the Mean Field Game, the HJB equation (4.72). The second equation, the FPK equation models the evolution of the system and is then defined:

$$\partial_t m(t,X) - B\log_2\left(\frac{Bh\partial_Q v^*(t,Q)}{log(2)(\sigma_n^2 + I(t))} \right) \partial_Q m(t,X) - m(t,X)\partial_Q \omega(t,X,p) = 0 \tag{4.73}$$

With the initial users states density, $m_0(X) = m(0, X)$, known.

Also, we have $\partial_Q \omega(t, X, p) = \frac{Bh}{log(2)} \frac{\partial_{QQ} v^*(t,X)}{\partial_Q v^*(t,X)}$. And the interference term is then calculated as:

$$I(t) \approx \left(\int_{h_{int} \in \mathcal{H}} \theta(t, h_{int}) h_{int} dh_{int} \right) \left(\int_{h \in \mathcal{H}} \int_{Q \in \mathcal{Q}} m(t,X) p^*(t,X) dh dQ \right) \tag{4.74}$$

4.7.4 Simulation Results

4.7.4.1 Simulation Parameters and Performance Criterion

In this section, we investigate the performance of the three previously detailed strategies. The performance criterion to be considered for each strategy under investigation, is again the average cumulated power cost per user E, from Equation 4.65. In order to observe the average performance of each strategy under investigation in terms of cumulated energy cost per user, we run Monte-Carlo simulations with $N_{MC} = 1000$ independent iterations. The parameters used for simulation are the same as before, listed in Table 4.1. But we now consider that the channels $\bar{h}_{ij}$ are uniformly selected in $\mathcal{H} = [h_{min}, h_{max}]$, with $h_{min} = 0.1$ and $h_{max} = 1$. A resolution of 20 elements is considered for $\mathcal{H}$.

4.7.5 Analysis of the Mean Field Equilibrium and the Mean Field Strategy

In this section, we present the graphs related to the Mean Field evolution $m(t, X)$, as well as graphs representing the Mean Field power strategy $p^*(t, X)$, for a single Monte-Carlo realization. Let us first focus on the evolution of the population represented in Figure 4.12. In Figure 4.12, we represented the evolution of the packet sizes $\bar{m}(t, Q) = \int_{h\in\mathcal{H}} m(t, Q, h)dh$. At t = 0, we observe an initial uniform distribution of users over the set of packet sizes $Q(0)$. At $t = T$, almost every user has achieved its transmission as requested, only a little proportion of users have a very small remaining packet to receive, therefore accepting the penalty function. This suggests an appropriate choice of the penalty function. In Figure 4.13, we present the Mean Field power strategy $p^*(t, X)$ used when $h = 0.5$. It appears, as before in Section 4.6.4, that the instantaneous power strategy $p^*(t, X)$ to be used at time t assuming $Q(t) = Q$ increases when the remaining packet size increases, but also increases when the system gets closer to the deadline. In Figure 4.14, we present the Mean Field power strategy $p^*(t, X)$ used when $t = \frac{T}{2}$. It appears that the the instantaneous power strategy $p^*(t, X)$ to be used at time t assuming $Q(t) = Q$ increases when the channel h decreases: this illustrates that the system adapts the power to be used to the channel h.

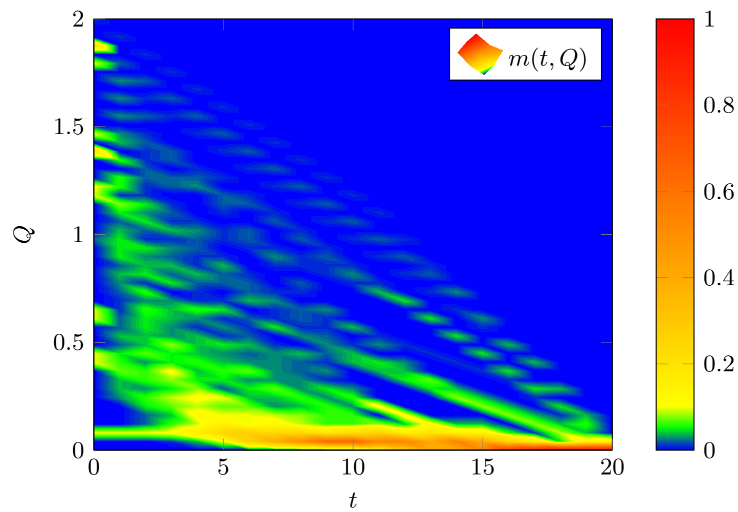

Figure 4.12: Optimal distribution of users $\bar{m}(t, Q)$ whose packet size at time slot t (x-axis) is Q (y-axis). Channel model 2.

4.7.6 Performance Analysis of the Investigated Strategies

In this section, we investigate the behavior of the three strategies under investigation for a single Monte-Carlo realization. In the following we investigate the behavior of three users with different initial packet sizes and channels and study the performance of the three power strategies under investigation for each user. The users under investigation, are:

- *User 1:* $Q_i(0) = 50$ and $\bar{h}_{ii} = 0.5$.
- *User 2:* $Q_i(0) = 50$ and $\bar{h}_{ii} = 0.1$.
- *User 3:* $Q_i(0) = 100$ and $\bar{h}_{ii} = 0.1$.

In Figure 4.15, we represent the instantaneous power strategy used by every single user in each strategy under investigation. Again, it appears that the Mean Field Strategy and the Constant power strategy are closely related. The instantaneous consumed powers increase with the initial packet size $Q_i(0)$ increases and increase when the channel $\bar{h}_{ii}$ decreases. This means that the *Mean Field* strategy is able to approach notably the optimal power strategy, which is given by the *Constant Power* strategy, as detailed in Section 4.7.2. As expected, the *Full Power* strategy transmits at full power, in order to complete the transmission as soon as possible. In Figure 4.16, we present the packet size

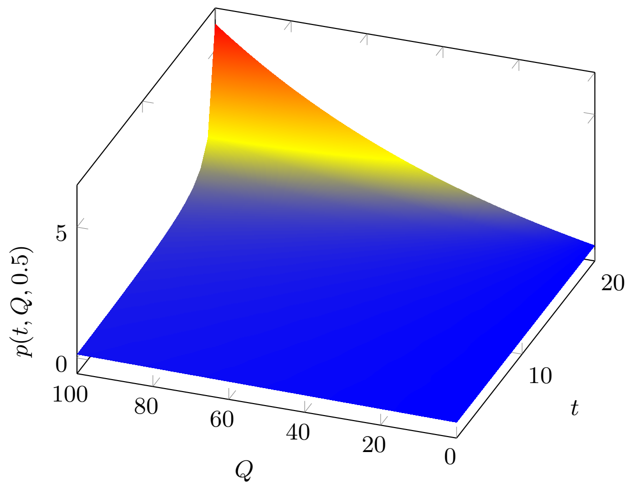

Figure 4.13: Optimal instantanous Mean Field power strategy $p(t, Q)$ to be used by any user whose packet size at time slot t (x-axis) is Q (y-axis), by a user whose channel is $h = 0.5$. Channel model 2.

evolution for our 3 users, and observe that the optimal strategy transmits the same amount of packet on every single time slot, and so does the *Mean Field* strategy. Both strategies are then strictly equivalent to an equal-bit scheduler, as previously discussed in Section 4.6.2. The *Full Power* strategy on the other hand is unable to schedule its transmission over the latency of T time slots offered to the system. We can observe on Figure 4.17 the cumulated power costs for each user and each strategy. We observe that the *Mean Field* strategy has a cumulated power cost close to the optimal *Constant Power* strategy, whereas the *Full Power* strategy has a poor energy efficiency. This demonstrates that the Mean Field is a good approximation of the optimal strategy to be used by any user in the system. This also highlights a significant performance gain, compared to *Full Power* strategies, which is simply due to the energy efficiency vs. latency trade-off. In order to observe the significance of the potential gain offered by an optimal strategy compared to a state-of-the-art *Full Power* strategy, we represented on Figure 4.18, the histogram of average Energy Cumulated Cost per user, over the N_{MC} Monte-Carlo realizations, for each strategy under investigation. Again, we observe that the *Mean Field* and the *Constant Power* strategies have close performance. We also observe a significant gain compared to a *Full Power* strategy. On average, the *Mean Field* and the *Constant Power* strategy allows for a reduction of the cumulated energy cost per user of 75.2%

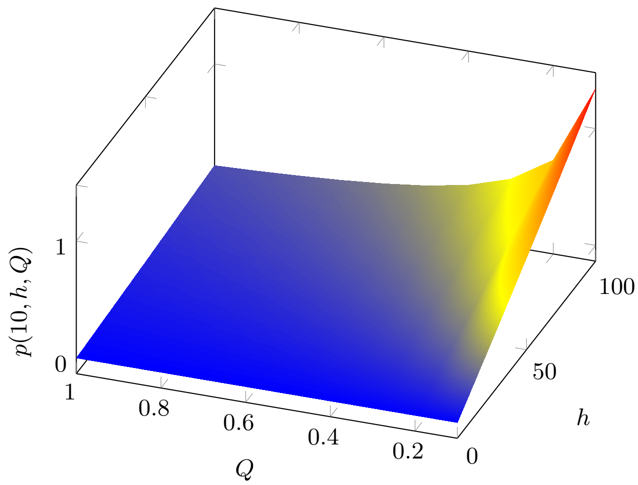

Figure 4.14: Optimal instantanous Mean Field power strategy $p(h, Q)$ to be used by any user whose packet size at time slot $t = 10$ is Q (y-axis) and whose channel is h (x-axis). Channel model 2.

compared to the *Full Power* strategy. The presented simulations highlight the significance of the potential performance gain that can be obtained by exploiting the energy efficiency vs. latency trade-off, and also shows that the *Mean Field* strategy closely approaches the optimal performance. However, there is no gain, in knowing the future, since the optimal power strategy could have been computed using an unaware scheduler, namely the equal-bit scheduler. There will be a gain, obtained by exploiting future knowledge, when the channel becomes time varying, which is detailed in the following section.

4.8 Channel Model 3: Time-Varying Channels

4.8.1 Introduction and Optimization

In this section, we assume that the channel are now time-varying, but no stochasticity is yet considered. The system now has to adapt the instantaneous power strategies to the channels variations. Our objective in this section consists of finding the set of optimal power strategies p^*, as the set of the optimal power strategies of each user i, denoted p_i^*, solution to the simplified version of the

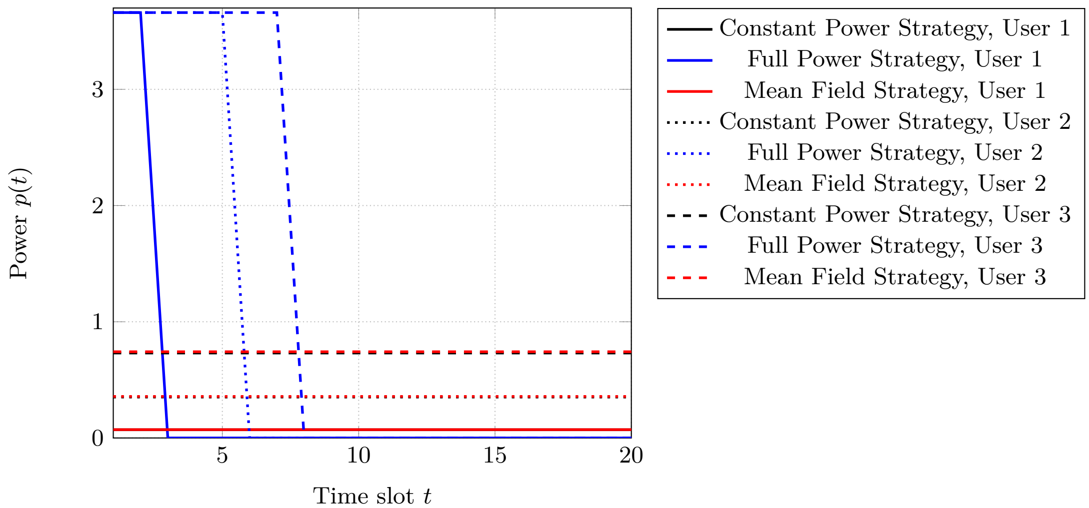

Figure 4.15: Instantaneous power strategies $p(t)$ for 3 strategies, 3 different users. Channel model 2.

optimization problem (4.8), defined hereafter:

$$\begin{aligned} & p_i^* = (p_i^*(1), ..., p_i^*(T)) = \arg\min_{p_i} \left[\mathbb{E} \left[\sum_{t=1}^{T} p_i(t) \right] \right] \\ & \text{s.t. } Q_i(t) = Q_i(t-1) - Blog_2 \left(1 + \gamma_i(t)\right) \Delta_t \\ & \text{and } dh_{ij}(t) = \alpha_{ij}(t) dt \\ & Q_i(T) = 0; \end{aligned} \tag{4.75}$$

Where

$$\gamma_i(t) = \frac{h_{ii}(t) p_i(t)}{\sigma_n^2 + \frac{1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^{N} h_{ji}(t) p_j(t)} \tag{4.76}$$

And the channel variations have been arbitrarily modeled $\forall i, j \in \mathcal{N}^2, \forall t \in \mathcal{T}$, as follows:

$$h_{ij}(t) = C_0 sin(f_0 t \Delta_t) + h_{ij}(0) \tag{4.77}$$

The constant power strategy is no longer the optimal power strategy, since there are now time variations on the channels. Computing the optimal power strategy can still be done using the iterative time water-filling algorithm we described in Section 4.3.1. This channel evolution models up and down states for the channel with a frequency f_0 and an amplitude C_0, as demonstrated on Figure 4.19.

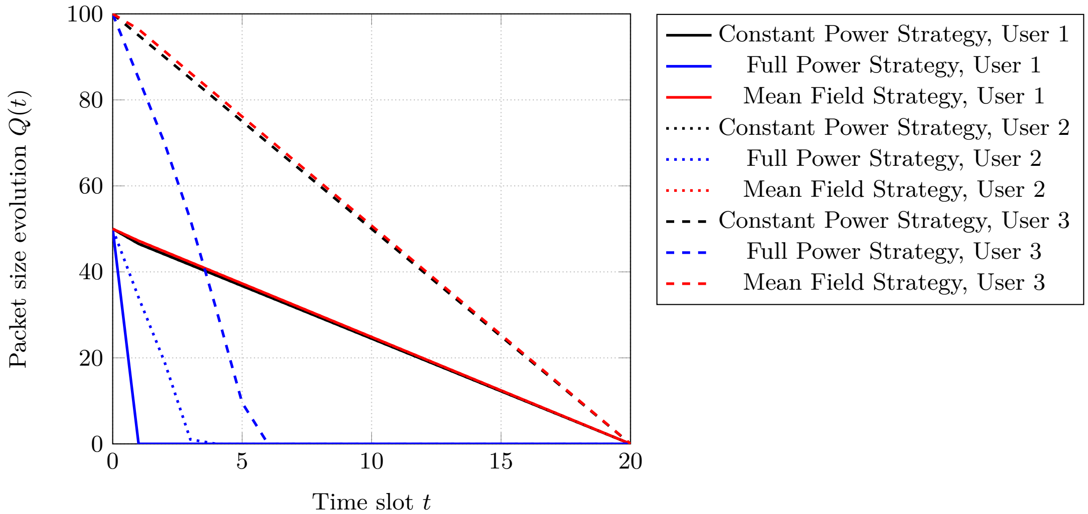

Figure 4.16: Packet Sizes Evolutions $Q(t)$ for 3 strategies, 3 different users. Channel model 2.

4.8.2 Updated MFG PDEs

In this scenario, we have $\alpha_{ij}(t,h_{ij}) = C_0 f_0 \Delta_t - cos(f_0 t \Delta_t)$, $\alpha(t,h) = C_0 f_0 \Delta_t - cos(f_0 t \Delta_t)$ and $\sigma_b^2 = 0$. The channel h appears among the state variables to be considered in the MFG PDEs, as well as the partial derivatives wrt. h. The optimal running cost trajectory v^* and power p^* are functions of t and $X = (h, Q)$.

$$\begin{aligned} p^*(t,X) &= \arg\min_{p(t,X)} \left[p(t,X) + \alpha(t,h)\partial_h v^*(t,X) - \omega(t,X,p)\partial_Q v^*(t,X) + \frac{1}{2}\sigma_b^2 \partial hhv^*(t,X) \right] \\ &= \tfrac{B}{log(2)} \partial_Q v^*(t,X) - \tfrac{\sigma_n^2 + I(t)}{h} \end{aligned} \tag{4.78}$$

The HJB equation can be simplified as:

$$\forall t, \forall X, \frac{\sigma_n^2 + I(t)}{h} + \alpha(t,h)\partial_h v^*(t,X) - \left[\frac{B}{log(2)} - B\log_2\left(\frac{Bh\partial_Q v^*(t,X)}{log(2)(\sigma_n^2 + I(t))} \right) \right] \partial_Q v^*(t,X) + \partial_t v^*(t,X) = 0 \tag{4.79}$$

With the final condition $v^*(T, X(T)) = K(Q(T))$

We have then defined the first fundamental equation for the Mean Field Game, the HJB equation (4.72). The second equation, the FPK equation models

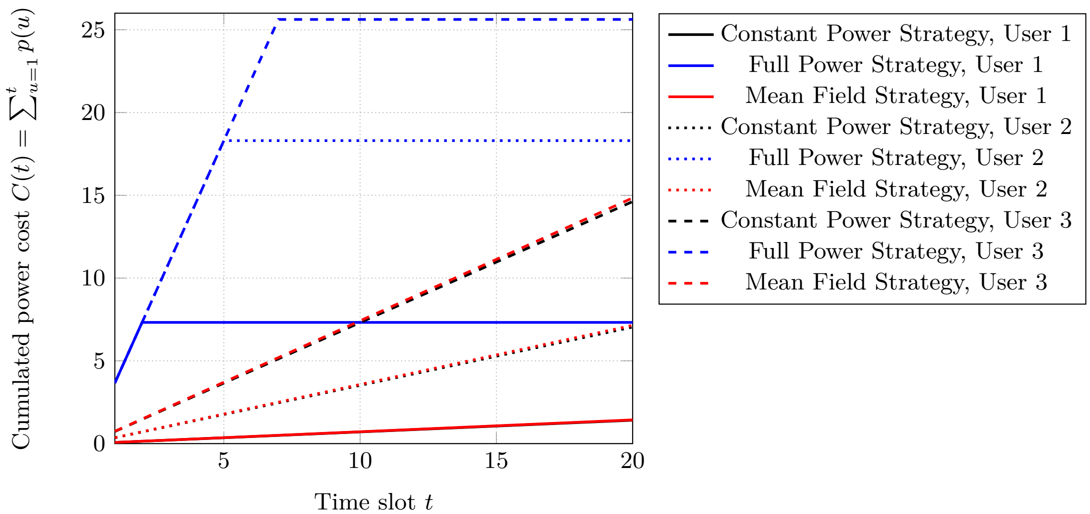

Figure 4.17: Cumulated power cost $C(t) = \sum_{u=1}^{t} p(u)$ for 3 strategies, 3 different users. Channel model 2.

the evolution of the system and is then defined:

$$\partial_t m(t,X)+\alpha(t,h)\partial_h m(t,X)+m(t,X)\partial_h\alpha(t,h)-\omega(t,X,p)\partial_Q m(t,X)-m(t,X)\partial_Q\omega(t,X,p) = 0 \tag{4.80}$$

With the initial users states density, $m_0(X) = m(0,X)$, known. Also,

$$\partial_Q\omega(t,X,p) = \frac{B}{log(2)}\frac{\partial_{QQ}v^*(t,X)}{\partial_Q v^*(t,X)} \tag{4.81}$$

And the interference term is calculated as:

$$I(t) \approx \left(\int_{h_{int}\in\mathcal{H}} \theta(t,h_{int})h_{int}dh_{int}\right)\left(\int_{h\in\mathcal{H}}\int_{Q\in\mathcal{Q}} m(t,X)p^*(t,X)dhdQ\right) \tag{4.82}$$

4.8.3 Simulation Results

4.8.3.1 Simulation Parameters and Performance Criterion

In this section, we investigate the performance of the three previously detailed strategies. The performance criterion to be considered for each strategy under investigation, is again the average cumulated power cost per user E, from Equa-

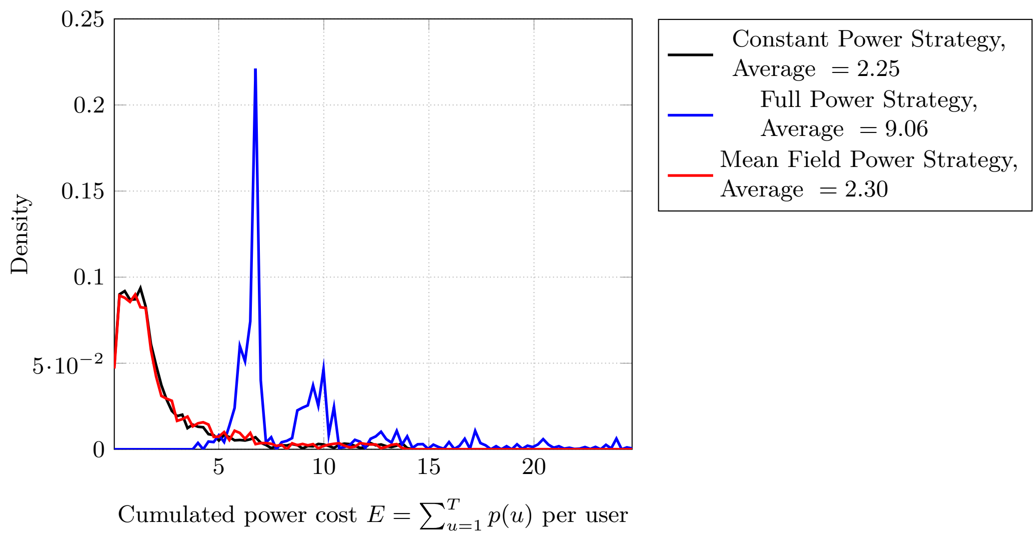

Figure 4.18: Histogram of the final cumulated power cost $E = \sum_{t=1}^{T} p(t)$ over the N_{MC} independent Monte-Carlo Realizations, for the 3 strategies. Channel model 2.

tion 4.65. In order to observe the average performance of each strategy under investigation in terms of cumulated energy cost per user, we run Monte-Carlo simulations with $N_{MC} = 100$ independent iterations: the number of users has been reduced, so that the constant power iterative algorithm could run in an acceptable computation time. The parameters used for simulation are the same as before, listed in Table 4.1. But we now consider that the channels initial channel values $h_{ij}(0)$ are uniformly selected in $\mathcal{H} = [h_{min}, h_{max}]$, with $h_{min} = 0.1$ and $h_{max} = 1$. A resolution of 20 elements is considered for $\mathcal{H}$. The time variations of the channels are conditioned by $C_0 = 0.3$ and $f_0 = 1000$.

4.8.4 Analysis of the Mean Field Equilibrium and the Mean Field Strategy

In this section, we present the graphs related to the Mean Field evolution $m(t, X)$, as well as graphs representing the Mean Field power strategy $p^*(t, X)$, for a single Monte-Carlo realization. We present here the optimal power strategies $p^*(t, X)$ to be used in three configurations of the channels:

- In Figure 4.20, the channel is in a low state for every user of the system, the

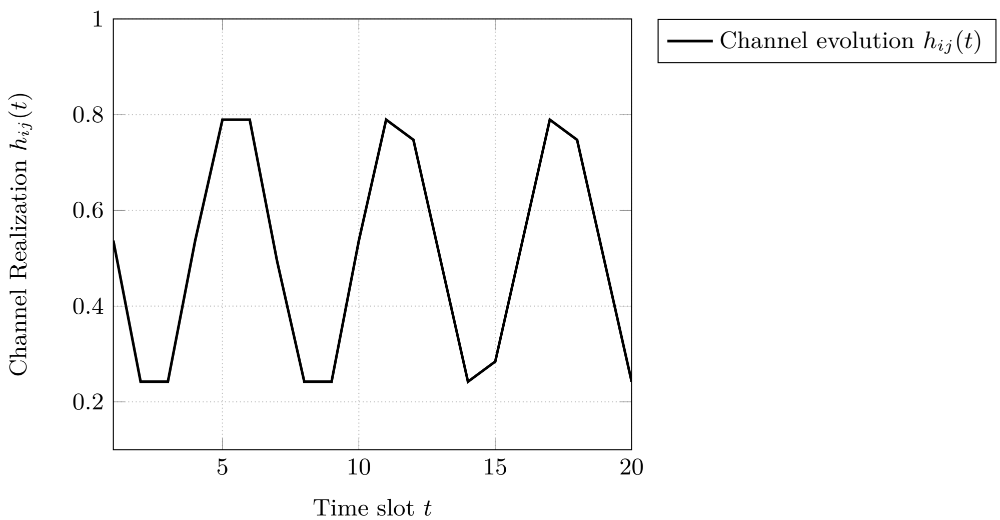

Figure 4.19: Channel evolution $h_{ij}(t)$, with parameters $C_0 = 0.3$ and $f_0 = 1000$, resolution of 20 elements for $[1, T]$, T = 20. Channel model 3.

power strategy reveals that the system only transmits on users that both had a high initial channel $h_{ij}(0)$ and still have a large packet remaining at $t = 9$.

- In Figure 4.21, the channel is in a high state for every user of the system, the power strategy reveals that the system transmits on every user large.
- In Figure 4.22, the channel is in a low state for every user of the system, the power strategy reveals that the system only transmits on users that both had a high initial channel $h_{ij}(0)$ and still have a large packet remaining at $t = 16$. However, since the system is closer to the deadline, more users are inclined to transmit, due to the urgency of the approaching deadline.

In all three presented strategies, we can observe that the power is adapted to the remaining packet size and the channel values: the higher the channel, the lower the power, the higher the packet remaining, the higher the power. It also appears a threshold on both h and Q where the system does not transmit as it estimates that the channel h is too bad and/or the packet remaining is low enough so that it could wait the next time slots to transmit, as it would actually do in a time water-filling algorithm. For this reason, it appears, that the optimal power strategy returned by the Mean Field strategy, is able to take into account the channel evolutions and adapts the power in adequation to the channel evolutions.

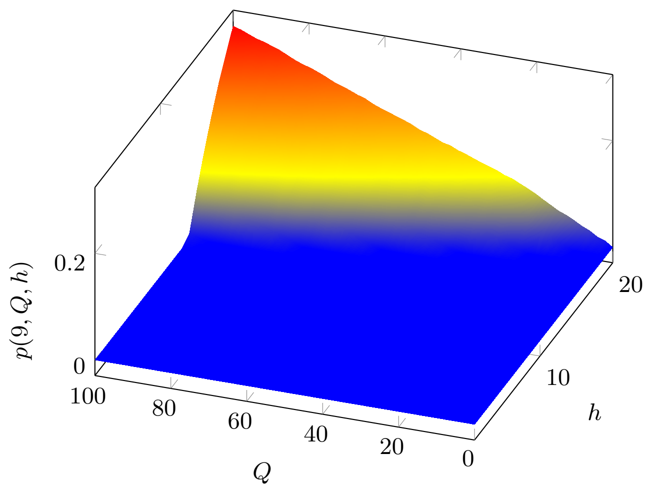

Figure 4.20: Optimal instantanous Mean Field power strategy $p(h, Q)$ to be used by any user whose packet size at time slot $t = 10$ is Q (y-axis) and whose channel is h (x-axis), at time slot $t = 9$ (Poor channel realizations, far from deadline). Channel model 3.

4.8.5 Performance Analysis of the Investigated Strategies

In this section, we investigate the behavior of the three strategies under investigation for a single Monte-Carlo realization. In the following we investigate the behavior of three users with a single initial packet size $Q(0) = 100$ and an initial channel $h(0) = 0.5$. In Figure 4.23, we represent the instantaneous power strategy used by our user in each strategy under investigation. It appears that the *Mean Field* strategy is the only one able to take into account the channel variations, as we can observe by comparing the presented graph to the channel evolution, presented in Figure 4.19. There is a slight difference between the three strategies, and again, the *Mean Field* strategy is the only one able to adapt the rate to the channel variations. We can observe on Figure 4.24 the cumulated power costs for each strategy. We observe that the *Mean Field* strategy has a cumulated power cost notably lower than the optimal *Constant Power* strategy, whereas the *Full Power* strategy has a poor energy efficiency. This demonstrates that the *Mean Field* strategy improves the power efficiency of the system, compared to the 2 other reference strategies. This also highlights

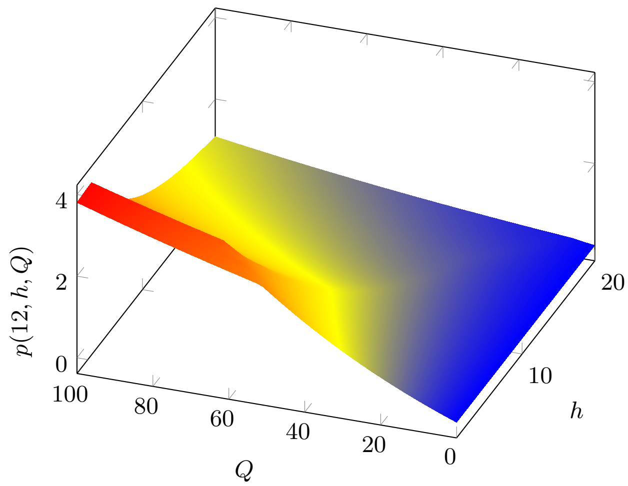

Figure 4.21: Optimal instantanous Mean Field power strategy $p(h, Q)$ to be used by any user whose packet size at time slot $t = 10$ is Q (y-axis) and whose channel is h (x-axis), at time slot $t = 12$ (Good channel realizations, far from deadline). Channel model 3.

a significant twofold performance gain. First, there is a gain due to the energy efficiency vs. latency trade-off. Second, there is a gain, obtained obtained when taking into account the future knowledge. It should also be noted that the MFG power strategy appears to transmit at a notably higher power when it gets close to the deadline: this phenomenon is due to the penalty function, which has to be balanced. In our case, the penalty function was slightly not penalizing the system enough, which is the reason why it did not transmit as it should at the beginning and had to rush the transmission at the end. We do not detail here how the penalty function is adapted to the simulation parameters.

In order to observe the significance of the potential gain offered by an optimal strategy compared to a state-of-the-art *Full Power* and *Constant Power* strategies on the whole set of users, we represented on Figure 4.25, the histogram of average Energy Cumulated Cost per user, over the N_{MC} Monte-Carlo realizations, for each strategy under investigation. There appears a first gain, of 62.1%, due to the latency, that can be observed between the *Full Power* and *Constant Power* strategies. There is also a second gain, this time, due to the capability of the system to exploit the prior future knowledge, in order to adapt the power settings to the channel evolution. This second gain corresponds to the performance gap between the *Constant Power* and the *Mean Field* strategies.

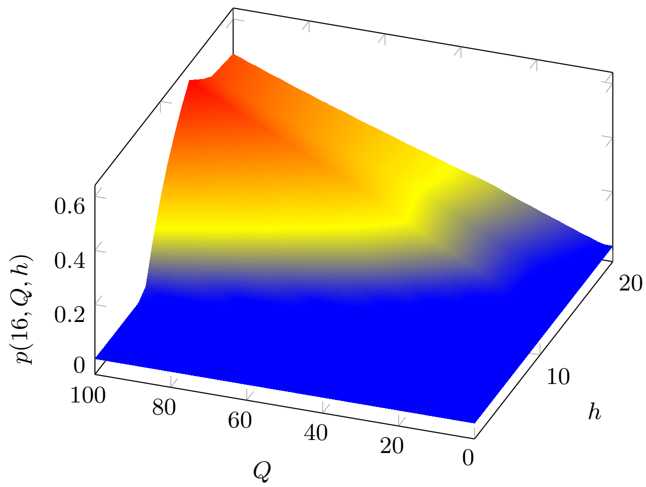

Figure 4.22: Optimal instantanous Mean Field power strategy $p(h, Q)$ to be used by any user whose packet size at time slot $t = 10$ is Q (y-axis) and whose channel is h (x-axis), at time slot $t = 16$ (Poor channel realizations, close to deadline). Channel model 3.

This additional gains reaches 19.6%, on average.

4.9 Channel Model 4: Stochastic Channels

In this section, we assume that the channels are time-varying with a similar evolution model as in Section 4.8. However, we consider that the Mean Field strategy now includes a non-zero stochastic term ($\sigma_b^2 > 0$). This stochastic term is used to model the uncertainty of the system about the future. In practice, we have to solve the complete MFG equations as they were listed in Section 4.5.2. In the following numerical simulations, we consider a single realization of the same game, with the same simulations parameters as in Section 4.8, and compare the performance of 3 strategies:

- The *Mean Field strategy, with no stochasticity* ($\sigma_b^2 = 0$), i.e. the system has perfect knowledge about the channels evolution: this power strategy is assumed to be the optimal one.
- The *Mean Field strategy, with low stochasticity* ($\sigma_b^2 = 0.05$)
- The *Mean Field strategy, with high stochasticity* ($\sigma_b^2 = 0.2$)

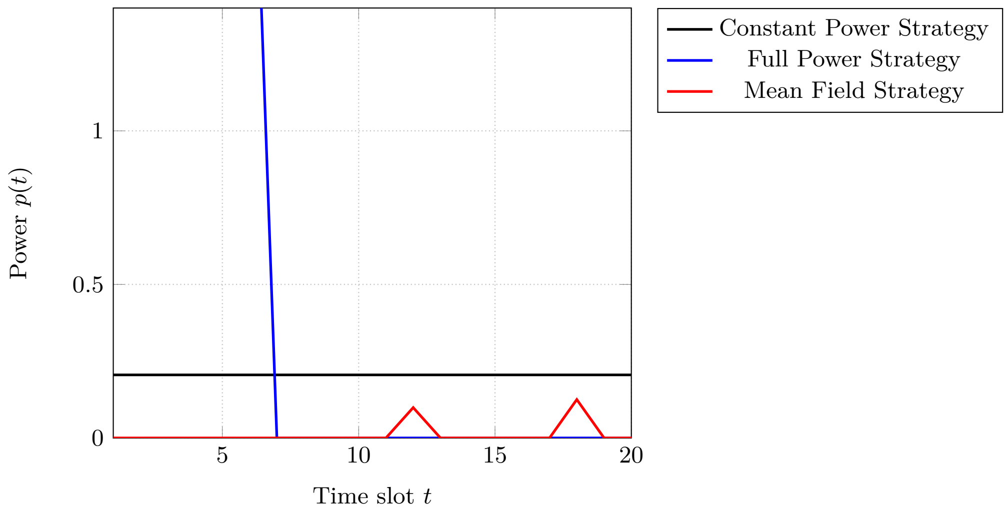

Figure 4.23: Instantaneous power strategies $p(t)$ for 3 strategies, initial packet sizes $Q(0) = 100$, initial channel $h(0) = 0.5$. Channel model 3.

We compare in Figure 4.26, the 3 power strategies for a user whose initial state is $Q(0) = 100$ and $h(0) = 0.5$. We observe that the uncertainty of the future badly affects the *Mean Field* power strategy: when the future becomes uncertain, the system tends to becomes more cautious and then transmits in advance compared to the*Mean Field* power strategy with no uncertainty. The system does so, in order to prevent a large packet to remain, in what seems to be an uncertain future. As a consequence, this tends to degrade the performance of the *Mean Field* power strategy. When the uncertainty is low (low value of σ_b^2), the system only transmits slightly in advance, thus slightly degrading the performance of the power strategy. However, when the stochastic term becomes dominant, the system becomes so uncertain about the future, that it transmits notably in advance. In that sense, it has a comparable behavior to the *zero knowledge* strategy from Section 3.4.2, which simply assumed that the future was the worst possible one, and thus rushed the transmission, becoming unable to exploit the offered latency. However, we do not investigate, in this thesis, the exact impact of the stochastic coefficient on the performance degradation of the *Mean Field* strategy.

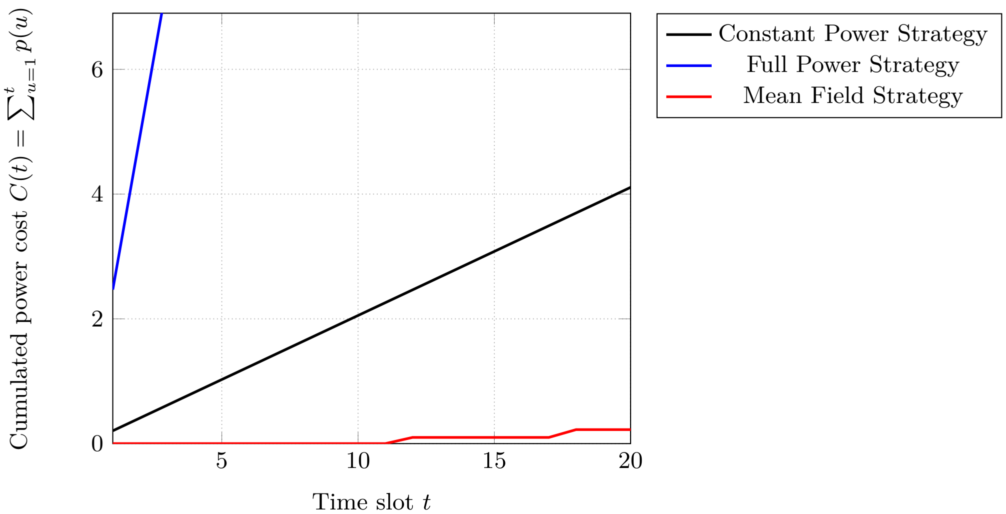

Figure 4.24: Cumulated power cost $C(t) = \sum_{u=1}^{t} p(u)$ for 3 strategies, initial packet size $Q(0) = 100$, initial channel $h(0) = 0.5$. Channel model 3.

4.10 Conclusions, Limits and Future Works

In this chapter, we have investigate the extension to a multiuser case of the proactive delay-tolerant problem detailed in Chapter 3, assuming the users were given perfect knowledge of the future transmission settings. The problem is modeled as a multiuser non-cooperative stochastic game, for which we conduct an analysis of the optimal configuration, namely the Nash Equilibrium of the game. We prove that the game rapidly became too complex to solve when the number of users in the system N became large. In order to deal with the complexity of such games, we identify three classical approaches used in literature. First, we propose an iterative time water-filling algorithm, capable of approaching the Nash Equilibrium, which sequentially adapts the transmission strategies of every single user, and updates the interference patterns after each power adaptation. However, we observe that such a process can only be used, with acceptable computation times, in scenarios where the number of users remains strongly limited. A first approach consists of computing the optimal strategies, in scenarios when the number of users N remains limited. A second approach consists of simplifying the problem by considering for example, simple channel models, with slow time variations. In these simple scenarios, the optimal solu-

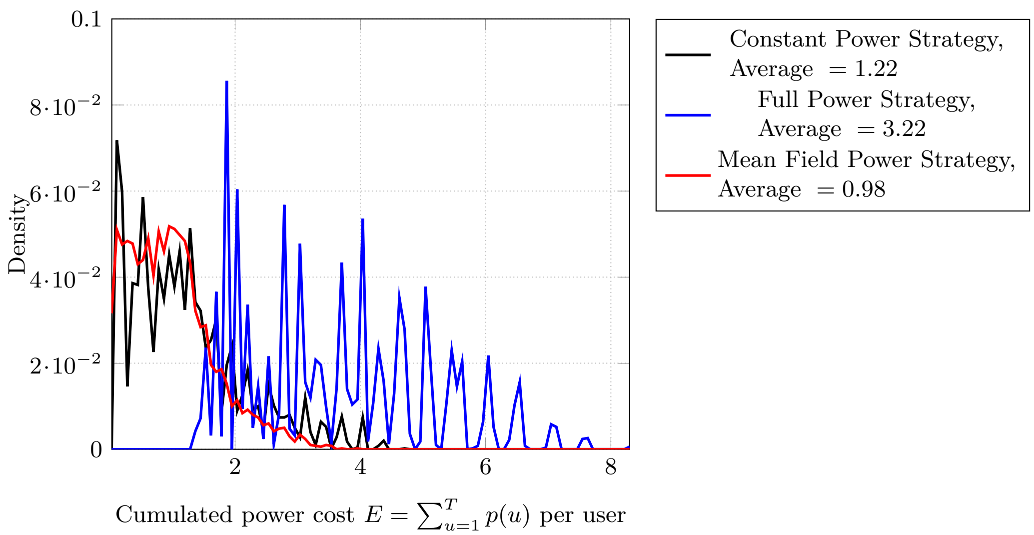

Figure 4.25: Histogram of the final cumulated power cost $E = \sum_{t=1}^{T} p(t)$ over the N_{MC} independent Monte-Carlo Realizations, for the 3 strategies. Channel model 3.

tion can be simply computed, by solving sets of N linear equations. However, when the channel model includes time variations, . In order to help the iterative algorithm to converge faster, we also propose a suboptimal iterative process, identical to the time water-filling one, but with an additional constraint of constant power. This additional constraint, leads to optimal solutions in constant channel scenarios. However, when there are time variations on the channels, the proposed iterative constant power algorithm becomes suboptimal, as it is unable to take into account the channel variations as a time water-filling algorithm would. This algorithm can still be used to obtain a good heuristic strategy, as it is capable to take into account the latency offered to the system. Often simple to compute, such heuristic strategies can be used, as a third option, but come at the cost of suboptimality. Another example of heuristic we detailed in this chapter consisted of the full-power strategy, where every user transmits at a notably high power, and rushes the transmissions. This strategy is however unable to take into account neither the offered latency, nor the time variations of the channels.

In this chapter, we propose a Mean Field approach, based on Mean Field Theory, that can be used to analyze the multiuser stochastic non-cooperative

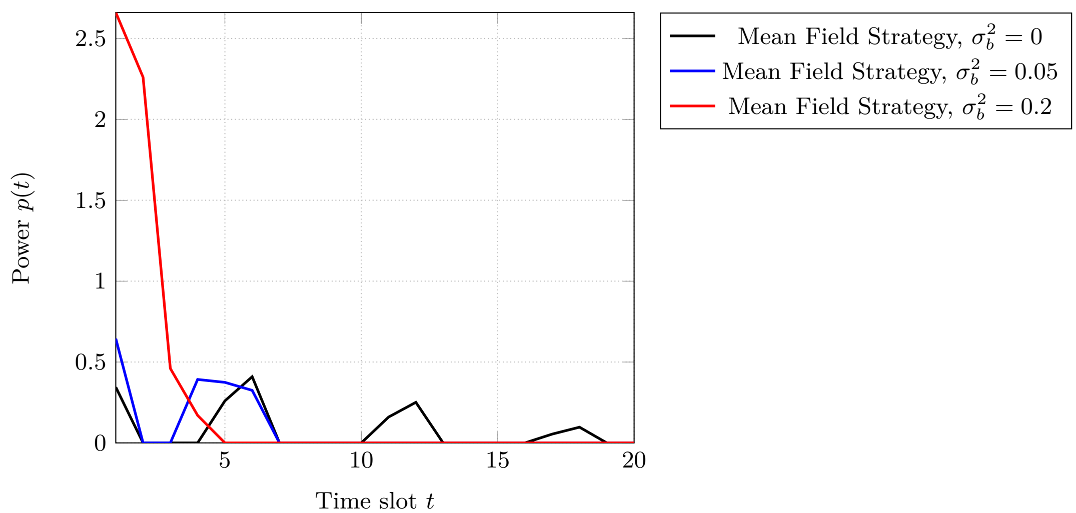

Figure 4.26: Instantaneous power strategies $p(t)$ for 3 strategies, initial packet sizes $Q(0) = 100$, initial channel $h(0) = 0.5$. Channel model 4.

game, by approximating the previous game: we transition from the previous N-users game into a Mean Field Game. This game has a significantly lower complexity, as it only consists of two bodies: a single user of interest and a mean field, assembling the $N-1$ other users. We conduct the analysis of the Mean Field Equilibrium, which is the equivalent concept to the Nash Equilibrium in a Mean Field Game. The conducted analysis reveals a set of two fundamental coupled partial differential equations, that had to be solved in order to analyze the Mean Field Equilibrium: the first one, the Hamilton-Jacobi-Bellman equation was used to compute the optimal power strategies, whereas the second one, the Fokker-Planck-Kolmogorov led to the trajectory and response of the mean field, when it implemented the optimal strategies. Furthermore, we propose an iterative process allowing to numerically approach the Mean Field Equilibrium. Strongly similar to the previous iterative time water-filling algorithm, it has a notably low computation cost, as it only consists of iterations over 2 bodies instead of N. Once the Mean Field Equilibrium is known, we can derive the optimal Mean Field power strategy, that can be used into the N-users stochastic non-cooperative game, as a good approximation of the optimal strategy, one could have obtained if we were able to compute the Nash Equilibrium of the N-users game. Finally, we provide numerical simulations, for different channel models, that give good insights about each strategy performance, including our Mean Field Game approach. Our conducted analysis revealed the following insights.

- In constant channel scenarios, it appears that the optimal power strategy can be simply computed by solving a set of N linear equations. Such a process can become rapidly overwhelming, when the number of users in the system N becomes large. However, our Mean Field approach leads to close-to-optimal results, with an additional advantage: it can be used at a fixed computation cost, whatever the initial number of users N is, as it transitions from any number of users N (supposed large) to a 2 bodies problem.

- When the channel model includes time variations, the optimal power strategy can no longer be analytically computed. The iterative time water-filling process rapidly becomes untractable, when the number of users becomes large. Adding a constant power constraint, allows to reduce the complexity of the iterative algorithm, but comes at the cost of suboptimality, as the algorithm is unable to take into account the channel variations of the system. This additional complexity, on the other hand, only slightly affects the Mean Field process, as it only complexifies the two fundamental PDEs with an additional partial derivative term.

- When stochasticity is included, the stochastic part plays the role of uncertainty for the system. As the system becomes more uncertain about the exact future channel realizations, it becomes more cautious and tends to transmit slightly more in advance, compared to what an optimal strategy would do, thus degrading slightly the performance of the Mean Field strategy to what could have been the optimal one. We have not investigated in this chapter, how the performance of the Mean Field strategy would degrade wrt the stochastic coefficient σ_b. The larger the σ_b coefficient, the more prominent, the uncertainty becomes. The extreme limit scenario consists of an unpredictable channel evolution for the system, as the stochastic part dominates the channel evolution. This will be investigated in future work.

- The global performance gain observed for our Mean Field strategy can be decomposed in two parts. First, it appears that there is a first significant gain due to the latency offered to the system: it is easy to visualize it by comparing the performance of our proposed Mean Field strategy, that is able to take into account the latency, to strategies that are not able to (e.g. the full power strategy). This is an expected result, as it is a simple illustration of the latency vs. energy efficiency trade-off. However, we also highlighted an additional gain due to the future knowledge: since our Mean Field strategy is able to take into account some piece of future knowledge about the channel variations, in addition to the latency, it may benefit from an additional gain, compared to strategies that can not (eg. the constant power strategy, or the full power strategy). However, for such a gain to exist, there must be time variations on the channels. Otherwise, the optimal power strategy can be computed analytically, through a simple set of N linear equations.

The presented results show insights on significant gains in terms of energy efficiency, offered by proactive delay-tolerant transmissions schemes. However, the presented works could be enhanced by taking into account several possible enhancements that we discuss hereafter:

- **More realistic channel model:** The system we considered is unrealistic on many points. For starters, we deliberately considered unrealistic arbitrary channel models, in order to observe specific behavior of both the Mean Field algorithm and the Mean Field strategy. The theoretical analysis provided insights on potential significant gains, that one could obtain by exploiting both the latency and perfect future knowledge. A necessary extension of the presented work requires a realistic channel model, as it is necessary to observe if the potential theoretical gains will scale when considering realistic channel models. The channel model we considered in the game definition, consisted of an auto-regressive process of order 1, with both a deterministic part (used to model an accurate prediction) and a stochastic part (used to model uncertainty). Such models have been used widely in Mean Field Games mathematical theory. But they suffer from a flaw, when we use them to model channels in the telecommunication field: due to the stochastic part, there is a non-zero probability that any channel $h(t)$ might go to infinity, when the time t tends to infinity as well. That is the reason why such an auto-regressive model can be questioned, when used to model channel evolution. For the moment, these models are the only ones for which we have theoretical results in Mean Field theory. Several ongoing works have however tried to extend the Mean Field Theory to different evolution models, as listed in [170].

- **MFG assumptions:** When transitioning to the Mean Field Game, we made an assumption on the users indistinguishability. This indistinguishability property allowed to regroup $N-1$ users into a mean field, thus simplifying greatly the system, but it also implies that the primary channels used for transmission in each AP-UE pair, have the same dynamics. Such a strong hypothesis can be questioned. Future work can include different classes of users, in order to model different behaviors of users (e.g. different types of mobilities), different types of APs, etc. When $M, M < N$ different classes of users are considered, the Mean Field Equilibrium analysis becomes more complex, as we must solve M HJB equations, in order to obtain the optimal power strategy to be used by every user, depending on the class it belongs to, as introduced by Nash in 1951 [171], each equation corresponding to the M different classes. In our analysis, we assumed only one class of users, thus leading to only one HJB equation, used for computing the optimal strategy to be used by every user in the system. an extension of the presented work, in heterogeneous networks for example, might require to consider classes of users. The extreme case with N classes of one user, leads to a set of N HJB equations, which is strictly equivalent to the analysis of the multiuser non-cooperative stochastic game.

- **Different utility function and sleep mode:** In the presented work, we have again assumed that the objective was to minimize a utility function consisting of the total transmission power. We could enhance the power consumption model by considering, for example, a more complete power consumption model, which takes into account the operating and primary costs of an AP, as suggested in [8]. If such a model was considered, less importance would be given to the transmission power costs and we would probably consider scenarios where the AP can be turned into idle mode, when unused for transmission, which is also a promising feature for power efficiency [13, 14, 15]. It could lead to a new class of strategies, able to take into account sleeping modes, whose investigation could be of interest.

- **Limited knowledge and distributed approach:** In this chapter, we assumed a perfect knowledge at each pair, of the system parameters and evolution. A distributed approach, with limited knowledge at each AP-UE pair can also be investigated, with an inspiring example in [81].

- **Acquiring future knowledge and cost of learning:** Again here, we have not questioned how these elements of future knowledge could be obtained. In particular, we must discuss the 'cost of learning', namely the equivalent power cost required in order to acquire some elements of future knowledge. Investigating this 'cost of knowledge' is a difficult task and still an open question in research at the moment: at the best of our knowledge, there are only a few limited works that are trying to explicit this 'cost of learning'. A few ideas could be found in here [137], even though it is not directly related to wireless networks. More works however focus on defining the 'cost of feedback' [138], namely the cost one has to pay to transmit a piece of information from a central unit in charge of establishing predictions to the AP that needs it. It is a matter of importance, since we need to confront this 'cost of learning' to the potential performance gain that the system could benefit from the acquired future knowledge.

- **Handover:** We considered that each UE remains assigned to the same AP and does not perform handovers at any moment. An extension of the presented work, into a scenario where one user might, because of its mobility perform handover, can be easily investigated. Several works have demonstrated that it is possible to predict a handover and define the next cell to be used [172, 173]. As long as we can define the channel evolution for each UE, even if it performs handover during at some point, we can compute the optimal power strategy, according to our process. Such an extension will be investigated in future work.

- **High performance heuristic strategies:** Our choice of heuristic strategies was simple. We modeled a full-power strategy, as a simple way to define a strategy that is unable to take into account neither the latency nor the channel evolution. Future work will also investigate more sophisticated heuristic strategies, that are more efficient, in terms of energy efficiency.

Chapter 5

Interference Classification and Interference Matching

5.1 Introduction

In order to cope with the scarcity of spectral resources, the network is becoming more and more dense and heterogeneous. In such dense heterogeneous networks, a bottleneck which is strongly affecting the network performance, is in-band interference, which is due to large numbers of interferers sharing spectral resources on a same geographical area. In interference limited networks, the classical way to process any interference is by treating it as an additive source of noise. For this reason, classical Radio Resource Management (RRM) approaches tend to reduce or avoid the interference, so that reliable transmissions can occur. However, such resource allocations mechanisms do not exploit one of the recent advances in information theory, namely interference classification (IC) [85]. According to interference classification, interference does not have to be necessarily avoided or strongly limited. Several techniques, for example Successive Interference Cancellation (SIC) techniques, allow the system to decode and cancel the interference signal, when the interference is strong compared to the useful signal. In this chapter, we propose a first paradigm shift: a novel interference processing aware RRM paradigm, that exploits IC, to enhance network performance. In particular, we focus on the short-term optimization of network performance. First, we investigate how the system can adapt the spectral efficiencies and how the interference is perceived by interferers, to the 2-users Gaussian Interference

Channel (2-GIC) configuration. The goal of the conducted optimization is to enhance the total spectral efficiency, after interference processing, of the system. We also investigate a second paradigm shift, which extends the use of this IC in a problem with multiple interferers per Access Point (AP), where the objective consists of matching interferers from different coalitions. Two interferers matched together will be exploiting IC: for this reason, we propose to form couples of 'friendly interferers', namely interferers that can efficiently process the interference coming from the 'friendly interferer' they have been matched to. We then propose an algorithm which forms groups of 'friendly' interferers from different coalitions, sharing spectral resources and thus interfering, in a way so that they can exploit interference according to IC. The optimal configuration is then defined as the joint matching of interferers, individual spectral efficiencies and interference regimes that maximize the total spectral efficiency of the system, whose performance is then compared to reference RRM techniques, where the interference might only be treated as noise, or avoided through orthogonalization.

The remainder of this chapter is organized as follows. After introducing the motivations, contributions and related works to it, we introduce the concept of IC in a preliminary section, in Section 5.2. In Section 5.3, we introduce the system model and optimization problem to be considered in the first half of the chapter. In this section, the objective is to define the optimal interference regimes and spectral rates, in a 2-GIC, so that the total spectral efficiency after interference processing is maximized. In Section 5.4, we extend the optimization problem, by considering coalitions of multiple interferers assigned to each AP. The objective in this section is to find the optimal one-to-one matching of interferers, that maximizes the total spectral efficiency after interference processing. In Section 5.4.4, numerical simulations provide good insights on the potential performance gain offered by both the IC and the interferers matching compared to reference RRM methods. In Section 5.5, we extend the IC and matching problem to the case of the M-users Gaussian Interference Channels (M-GIC). Two difficulties arise: i) defining the IC in M-GIC is complicated and still an open question in research, and ii) the matching problem becomes a Multidimensional Assignment Problem (MAP) which is NP-Hard. Nevertheless, we propose to investigate the matching problem in the noisy interference regime and detail in Section 5.5.3, two suboptimal algorithms, with low complexity, that can compute a satisfying matching. Numerical simulations providing insights on the potential performance gains are given in Section 5.5.4. Finally, Section 5.6 concludes the chapter, describes the limits of the presented work and introduces the next chapter.

5.1.1 Motivations and Related Works

In the previous two chapters, we have proposed to enhance the energy-efficiency of the network, for a given sum rate constraint, which consisted of completing a required transmission before a given deadline. More precisely, we proposed a novel approach, which coupled both the well-known latency vs. energy-efficiency trade-off and with a piece of additional knowledge provided to the system (namely, predictions about the future transmission context). We demonstrated that the combination presented a great interest, for enabling energy-efficient networks. However, defining the optimal transmission power strategies appeared complicated, and the complexity is due to the numerous interactions between users, that were modeled through interference. In this chapter, we focus on the dual problem of the previous optimization problem. Indeed, we propose to optimize the network total spectral efficiency, in a fixed and constrained power configuration. Since it appeared that the interference was the central phenomenon, limiting the performance of the system in optimization, we propose to exploit underexploited properties of the interference, via different interference processing techniques, in order to enhance the total performance of the network.

One of the pillar solutions for enhancing the future 5G wireless networks capabilities consists of the densification of heterogeneous networks [174, 175, 84]. These networks suffer from several fundamental bottlenecks, which strongly affects and limits the network performance: herein, we focus on one of these bottlenecks, the in-band interference. The scarcity of spectral resources forces cells to overlap and operate in a common geographic area and share common spectral resources, thus causing in-band interference, which may drastically affect the reliability and efficiency of the transmissions between Access Points (APs) and their assigned User Equipments (UEs). Common understanding is that interference, which is classically processed as additive noise, compromises the performance and must be ideally avoided or at least strongly limited, so that reliable transmissions may occur. In that sense, Radio Resource Management (RRM) are used to limit undesired effects of in-band interference. Therefore, a first approach forces interference to be strictly avoided by orthogonalizing transmissions. Such interference management is enabled by partial or full orthogonalization between competing interferers, as proposed by time division multiplexing, time sharing , frequency reuse or graph coloring [76, 77, 78]. Nevertheless, orthogonalization of transmissions over spectral resources drives the system to a drastic suboptimal spectral efficiency operating point.

When interference is treated as an additive source of noise and not avoided, it must remain limited, so that reliable and efficient transmissions can occur. To do so, the classical approach consists of carefully adapting the transmission powers, so that interference remains under a target limit. This is typically obtained through power control designs, where the system carefully balances transmission powers among interfering sources, as it was for example detailed in the previous chapter, or in multiple papers in literature [79, 10, 12, 11]. However, in order to solve such optimization problems and find optimal transmission strategies for every user in the system, we have to find an equilibrium configuration. When facing this problem, this may involve a high mathematical complexity, especially when the system dimensions become large [80, 81, 60], as we must take into account all the one user to one user interactions, which is modeled by the interference perceived at each receiver side. The problem consists not only in defining the equations that characterize the equilibrium of the power control game, but also in being able to analytically or numerically solve them. Commonly, in the power control designs consider an iterative algorithm to approach the equilibrium configuration, as suggested in Section 4.3.1, in which every user may adapt at each iteration its own power to the present interference pattern. The process is then repeated until a convergence criterion is reached. But, every time a user adapts its transmission power, it resets the overall network interference pattern and consequently all the interference patterns perceived by the other users. For this reason, power allocation techniques may suffer from high computational complexity and may not always converge to an equilibrium, due to 'ping pong effect' and users constantly re-adapting their power as detailed previously in Section 4.3 and in papers [60, 86].

However, the previously mentioned methods do not exploit the recent advances in the domain of information theory, showing that interference might not necessarily be an opponent, but may become, in fact, an ally, especially in cases where the interference becomes strong, which is commonly identified as an interference badly compromising the transmission. In practice, Carleial [82] and, later on, Han & Kobayashi [83] have demonstrated that it was possible to exploit intrinsic properties of the interference, in order to process interference differently and obtain notably higher rates after interference processing. The observation, which allowed the trick to happen, consisted of observing that interference was not just an additive source of noise. In fact, Carleial suggested that, in scenarios of strong interference, the strong limitation of the rate was not due to theoretical limitations, but was instead due to the communications techniques employed for processing interference. He proposed an interference processing technique, which first aims at decoding the strong interference in presence of the primary signal, and then subtract the decoded interference from the received signal, leaving it with no trace of interference. The main concept behind this idea is commonly referred to as Successive Interference Cancellation (SIC) and is considered as the optimal interference processing technique for strong interference scenarios, as it allows to remove completely the strong interference. Based on this observation, it immediately appeared that a single interference mitigation technique, namely the noisy processing, could not perform well for all the possible scenarios of interference, ranging from weak to strong interference.

As a matter of fact, exploiting additional interference mitigation techniques, such as SIC, has been recently perceived as a promising feature for 5G networks [84]. It also inspired the works of Etkin & Tse [85], who investigated the different interference mitigation techniques from a single user point of view and their spectral efficiencies after interference processing. They defined the SNR/INR configurations for which each interference mitigation technique was the most-suited technique. In a 2-GIC, they proved that for any pair (R_1, R_2) in the interference capacity region, the considered schemes were able to achieve the spectral efficiencies pair (R_1-1, R_2-1) for any values of the channel parameters (i.e. SNR and INR). Basically, this means that the presented schemes in this paper were able to achieve spectral efficiencies within 1 bit/s/Hz of the capacity of the interference channel. Five interference mitigation techniques were identified, each of them being the most-suited technique in a given region of α, which was defined as $\alpha = \frac{log(INR)}{log(SNR)}$. In the following we recall the presented classification of the interference mitigation techniques proposed by Etkin and Tse, as the '5-Regimes Interference Classification'. This '5-Regimes interference classification' was later simplified by Abgrall [86, 87], who proposed a simplified version of this classification, only including 3 interference regimes. The interference may either be treated as an additive source of noise if it is perceived as weak, exploited and canceled via SIC if it is perceived as strong, or simply avoided via orthogonalization, if it is neither perceived as strong or weak. To justify the simplification, Abgrall suggested that even if some of them perform very well theoretically, they may suffer from infeasibility in practice because of excessive computational complexity or strict operating assumptions. For example, simultaneous superposition coding is up to now too complex to be used in practice. For this reason, Abgrall favored the use of techniques which can be implemented in practical systems without stringent limitations. More details about each classification and the considered classification regimes will be detailed in a short tutorial, in Section 5.2.

Similarly to recent works that have proposed to exploit IC [88, 89], we propose an IC based approach, that enhances the system network performance. To do so, we first consider 2-GIC. The system deals with the perception of the interference at each receiver side and aims at maximizing the total spectral efficiency, assuming the interference is treated according to the 3-regimes classifier defined by Abgrall. More specifically, we propose a first paradigm shift: we propose to adapt the spectral efficiencies used for transmission to the channel context, in order to enforce the optimal interference mitigation technique to be used at each receiver side, thus maximizing the total spectral efficiency obtained after interference processing. This way, we reduce the complexity of the optimization problem, by only allowing changes on the interference perception of each user. We leave unchanged the short-term power configuration and interference patterns, since it causes an avalanche of changes in the network [60, 86]: such an approach directly tackles the 'ping pong effect' and the associated computational complexity observed in the iterative processes and instead allows for low-complexity optimization. The optimization problem conducted analysis reveals that, when maximizing the total spectral efficiency, interference does not have to be avoided. In fact, in this specific scenario, our study leads to a reduced IC, with 2 regimes for each user, that can be exploited in more sophisticated multiuser optimization problems. The two regimes are the noisy regime, used for weak interference scenarios and the SIC-based regime used in strong interference scenarios.

Moreover, we propose to exploit this new-built '2-regimes Interference Classification' in a matching problem, consisting of resource allocation among several interferers, sharing a common geographic area and spectral resources. Contrary to the previous scenario, we now consider $M \geq 2$ APs and M coalitions of $N > 1$

users assigned to each AP. Our objective is to form groups of interferers, i.e. N groups of M interferers, with exactly one UE from each coalition. Among the $N!^{M-1}$ combinations, we seek the optimal matching that maximizes the total spectral efficiency of the system. Users from different coalitions, belonging to a same group of interferers, share the same spectral resources, thus suffer from in-band interference, but may process it according to the classifier we previously defined, thus leading to the interference regimes and the maximal spectral efficiencies for each interferer in the group of interferers. First, for any given group of interferers, we investigate how the interference regimes are defined, i.e. we analyze the most efficient way to process interference at each receiver, so that spectral efficiency is maximized. To do so, we consider a M-GIC and assume that each transmitter has perfect Channel State Information (CSI) and may, at will and in coordination, adapt the interferers spectral efficiencies, thus changing the interferers regimes and enforcing the spectral efficiencies to be used by any interferer in any M-GIC, in order to maximize the total spectral efficiency after interference processing of the system. An analysis of this problem has been conducted for the 2-GIC in [116], which lead to a '2-Regimes Interference Classification'. We detail it more extensively in the Section 5.3. The second step of the optimization procedure, i.e. the second proposed paradigm shift, consists of finding the most appropriate matching, i.e. groups of interferers, such that all $N \geq 1$ users from each coalition are assigned to one and only one of the N groups of interferers, and the total spectral efficiency of the system is maximized. Mathematically speaking, the matching problem appears to be a multidimensional assignment problem (MAP). In the $M = 2$ case, the Kuhn-Munkres algorithm [96, 97, 98] is able to compute the optimal assignment in polynomial time.

At this point, the analysis of IC and matching is however limited to scenarios with coalitions of $M = 2$ interferers. Indeed, when the number of interferers coupled together becomes greater than two, defining the IC in a M-GIC becomes complicated and finding the best way to process interference at each receiver becomes rapidly impossible, as the number of regimes to be investigated skyrockets. Group SIC, iterative k-SIC or k-Joint Decoding approaches might be considered [90], leading to multiple new regimes, for which it is complicated to define the optimal interference processing procedure and a fortiori the spectral efficiencies after interference processing. Nevertheless, we study the matching problem in simple scenarios where interference is treated as noise or where orthogonalization is considered. Also, when $M > 2$, the MAP becomes NP-Hard [176] and finding the best matching becomes complicated, since there does not exist an algorithm which is both optimal and has a polynomial computation time. In order to approach the optimal matching, it is possible to formulate the problem as an Integer Linear Programming (ILP) problem, but the complexity of the branch-and-bound algorithms solving the ILP strongly increases when the number of coalitions M or the number of users per coalition N becomes large [177]. To tackle the complexity of such a matching problem, we consider, in this chapter, a memetic algorithm as in [99]. Even though suboptimal, such an algorithm is able to approach the optimal matching, in acceptable time, for large values of N or M. We also propose a suboptimal algorithm, which consists of layered Kuhn-Munkres optimizations. This algorithm has a remarkably low complexity and even though suboptimal, it has a notable performance.

In the end, we show that the system can exploit in-band interference, without modifying the short-term power allocation strategy, by both smartly coupling interferers from different coalitions and by defining the best way to process interference at each receiver side instead. The proposed algorithms present a low complexity and are able to exploit recent advances in interference management and classification. Numerical simulations show that the proposed RRM paradigm shifts offer significant spectral efficiency improvements compared to classical resource allocation algorithms, that are unable to exploit IC and/or are unable to perform interferers matching.

5.1.2 Contributions

The content of this chapter has been published in three papers. In the first paper [116], we study the IC in a 2-GIC and introduce our '2-Regimes Interference Classification'. In the second paper [117], we extend the previous analysis to a multiuser scenario (multiple users per AP and/or multiple APs) and introduce both concepts of 'friendly interferers' and interferers matching. This leads to a matching problem, that is extensively analyzed, eventually revealing the potential gains of this second paradigm shift. The third paper [112] is a journal paper, which sums up the contributions detailed in this chapter. The innovation and scientific contributions presented in this chapter are summed up as follows:

First, we investigate an optimization problem, including the recent IC advances. The objective of the conducted optimization is to define the optimal spectral efficiencies for a group of interferers, so that the total spectral efficiency of the group is maximized. The short-term power configuration is unchanged, as only the individual spectral efficiencies can be modified. As a consequence, this affects the perceived interference at each receiver side and we can define the optimal interference processing technique to be used to process interference. In the 2-GIC, the conducted analysis reveals that, in such an optimization, only two regimes prevail, thus leading to a low-complexity '2-Regimes Interference Classification' algorithm. This first paradigm shift reveals that interference regimes can be adapted to any 2-GIC configuration, to enhance the total spectral efficiency after interference processing. Second, we introduced a second paradigm shift, namely the concept of 'friendly interferers' matching, which consists of selecting the users that will be grouped together, and thus interfere. The previous IC results are then reused, as we assume that users belonging to a same group will adapt their spectral efficiencies so that they maximize their total spectral efficiency. The total problem is then modeled as a matching problem, which consists of forming the optimal groups of interferers, in order to maximize the total spectral efficiency of the system. An optimal algorithm is proposed for scenarios where only two APs and two coalitions of interferers have to be matched together. In a scenario where more than two APs and interferers coalitions are considered, the study of the IC rapidly becomes untractable, and the matching problem becomes NP-Hard. We leave for now the study of the M-GIC. It will instead be studied in detail in Section 6.4.2. Nevertheless, the matching problem, when interference is processed as noise exclusively, can be addressed. We then propose two efficient heuristics capable of computing a suboptimal solution to the matching problem, with polynomial computation time. The first algorithm is based on a memetic algorithm, used for solving Multidimensional Assignment Problems. The second one is inspired from an iterative Kuhn-Munkres procedure, and has significant performance, with low computation time. Numerical simulations benchmark the performance of the optimal matching, individual spectral efficiencies and interference regimes is computed and compared to the performance of classical reference RRM procedures, only implementing noisy interference processing or orthogonalization. The study provides insights on potential significant gains, that are offered by both the IC and the matching of interferers.

5.2 Preliminary on Interference Classification and Interference Regimes

Recent investigations in the domain of information theory have revealed that interference does not necessarily have to be strongly limited or ideally avoided, so that interference can be efficiently processed as noise without compromising the system performance. In fact, interference is not necessarily an opponent, but may become an ally in numerous configurations. Based on this, Etkin &Tse [85] investigated the 2-GIC and proposed a 5-regimes interference classifier, which determines the best way to process the incoming interference from one user, at the second user's receiver side. For any value of the criterion $\alpha = \frac{\log(INR)}{\log(SNR)}$, i.e. for any given SNR and INR perceived by a receiver, Etkin and Tse defined the interference mitigation technique that maximizes the theoretical spectral efficiency after interference processing, among all those detailed in literature at the moment. In this section, we provide a short tutorial on the IC concept, detailing the interference regimes considered throughout this chapter. Figure 5.1 sums up the results detailed in [85], revealing that the classification consists of 5 interference regimes: 5 interference mitigation techniques, 5 α regions of dominance for each interference mitigation technique and 5 theoretical maximal spectral efficiencies bounds $R(\alpha)$ obtained after interference processing. The theoretical maximal spectral efficiencies $R(\alpha)$ can be extracted from the degree of freedom r_α, which is defined, for large SNR values, as follows:

$$r_\alpha = \lim_{SNR \to \infty} \frac{R(\alpha)}{log(1+SNR)} \tag{5.1}$$

The classification into 5 regimes is detailed as follows, in Table 5.1.

Regime	Region of dominance	Scenario	Degree of freedom r_α	Rate after interf. process. $R(\alpha)$
Reg. 1	$0 < \alpha \geq \frac{1}{2}$	noisy interference	$(1-\alpha)$	$R(\alpha) \approx log(1+\frac{SNR}{INR})$
Reg. 2	$\frac{1}{2} \geq \alpha \geq \frac{2}{3}$	weak interference	α	$R(\alpha) \approx log(1+INR)$
Reg. 3	$\frac{2}{3} \geq \alpha \geq 1$	moderately weak interference	$(1-\frac{\alpha}{2})$	$R(\alpha) \approx log(1+\frac{SNR}{\sqrt{INR}})$
Reg. 4	$1 \geq \alpha \geq 2$	strong interference	$\frac{\alpha}{2}$	$R(\alpha) \approx log(1+\sqrt{INR})$
Reg. 5	$2 \geq \alpha$	very strong interference	1	$R(\alpha) \approx log(1+SNR)$

Table 5.1: Summary of 5 regimes

In the first regime, the interference is weak enough, so that it can be processed efficiently as an additive source of noise. Even though it is the simplest approach to treat interference, it was proven to be optimal, when no feedback

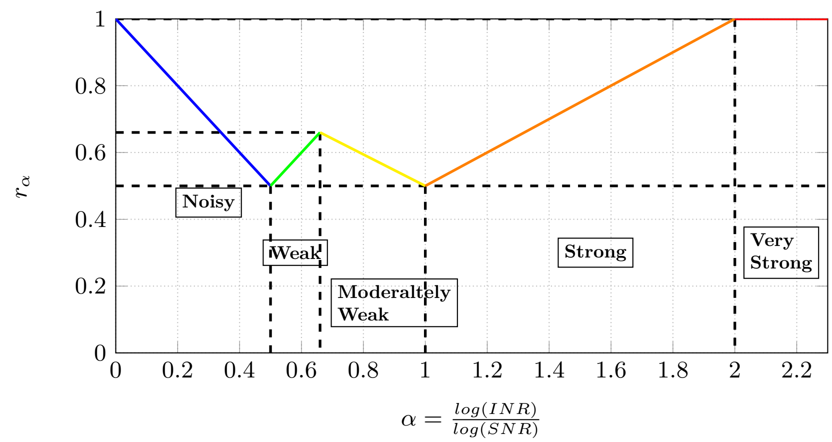

Figure 5.1: Generalized degrees of freedom, according to the α value. This 'W-shaped' curve exhibits an interference classification into 5 interference regimes.

about interference was available, in [178, 179, 180, 181]. The noisy strategy is however at fault when the INR nearly equals the SNR (i.e. $\alpha \approx 1$). More details about this regime are provided in Section 5.2.1. In the fifth regime, the interference is strong enough so that it can be decoded using Successive Interference Cancellation (SIC) techniques. The theoretical bound obtained after interference processing matches the point-to-point channel capacity, when no interference is present. More details about this regime can be found in Section 5.2.2. In between, 3 regimes coexist: the 3 in-between interference regimes considered in the 5 regimes classification, are based on both the theoretical studies made by Han & Kobayashi [83], commonly referred to as superposition coding and joint decoding, which extend the achievable capacity region, in comparison to previous works [182, 183].

This 5-Regimes interference classification was later simplified into a 3-Regimes interference classification and exploited for optimal power control under rate constraints by Abgrall [86, 87]. In the following section, we detail each one of the 3 interference regimes and its performance in terms of Spectral Efficiency after interference processing. We consider a 2-GIC, as depicted in Figure 5.2 and define the interference regime from the point of view of user 1. We denote O_i the interference mitigation technique (or interference regime) used by user $i \in \{1,2\}$ and $R_i(O_1, O_2)$, the maximal spectral efficiency after interference processing, for user i, assuming the two users interference regimes are respectively O_1 and O_2. We denote $\mathcal{O} = (O_1, O_2)$, the interference regime of the 2-GIC, which consists of the combination of the two individual interference regimes of the interferers. Finally, we denote γ_i, the SNR perceived by user i and δ_i the INR perceived by user i due to the transmission from user $j \neq i$. These notations are extensively detailed in the system model section, in Section 5.3.

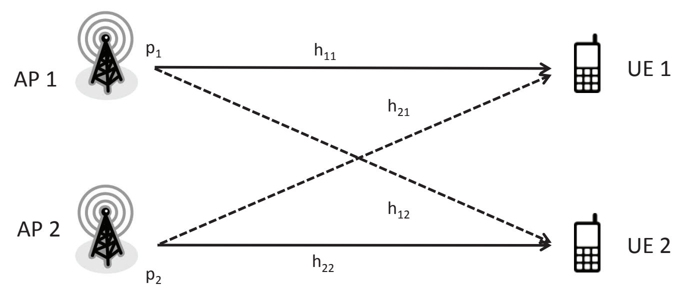

Figure 5.2: The 2-users Gaussian Interference Channel, considered during the first half of this chapter.

5.2.1 'Weak' Interference Regime: Interference as Noise

The first interference regime ($O_1 = 1$) is the noisy interference regime: the inband interference is not decoded by the receiver, but treated as an additional source of noise. Without loss of generality, let us focus on the case where the receiver is user 1. Such an interference processing technique is only efficient if the interference is experienced weakly, i.e. the SNR perceived at the receiver side γ_1 is high compared to the perceived INR δ_1. According to [85], this is equivalent to $\alpha_1 = \frac{log_2(\delta_1)}{log_2(\gamma_1)} < \frac{1}{2}$. We reformulate this constraint, as in [86], by stating that the user 1 must decode the incoming signal, in presence of noise and interference, which means that the channel must not be in outage and the maximal spectral efficiency for user 1 is then:

$$R_1(O_1, O_2) = \begin{cases} \log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right) & \text{if } O_2 \neq 2 \\ \frac{1}{2}\left[\log_2(1+\gamma_1)+\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)\right] & \text{if } O_2 = 2 \end{cases} \quad (5.2)$$

Note that $O_2 = 2$ corresponds to an interference regime we have not presented yet. It is detailed in Section 5.2.3.

5.2.2 'Strong' Interference Regime: SIC

The second interference regime ($O_1 = 3$) corresponds to the strong interference regime. In this regime, interference is processed with the SIC technique. The interfering signal is first decoded by processing the primary signal as an additive source of noise, then interference is subtracted and canceled out of the received signal. After processing the interference, the primary signal can be decoded, with no interference. Therefore, the performance reaches the point-to-point channel capacity. According to [85], this happens, when $\alpha_1 > 2$. As in [86], we reformulate this condition and state that user 1 must be able to decode the interference without outage, which is only possible if the spectral efficiency used by user 2 is low enough so that user 1 can decode interference through its interfering link, in presence of the primary signal, i.e.:

$$R_2(O_1, O_2) \leq \log_2\left(1+\frac{\delta_1}{1+\gamma_1}\right) \tag{5.3}$$

The user 1 must then decode the incoming signal, in presence of noise only (since interference has been canceled out by SIC), which immediately leads to a maximal spectral efficiency after interference processing for user 1, which correspond to the point-to-point channel capacity:

$$R_1(O_1, O_2) = \log_2(1+\gamma_1) \tag{5.4}$$

We can observe that such a SIC configuration involves a constraint on the maximal spectral efficiency of the interferer, which was not the case in the noisy regime.

5.2.3 The 'in-between' Interference Regime: Orthogonalization

The third interference regime ($O_1 = 2$) applies to cases where interference can not be decoded and is harming the transmission performance. According to [86], it corresponds to scenarios where $\frac{1}{2} < \alpha_1 < 2$. In such a context, we assume that the system can avoid the interference, if both users transmit using one half of the spectral resources, which is simply achieved through Time/Frequency Division Multiplexing. The point-to-point channel capacity is reached since no interference is present, but at a halved spectral efficiency for each user, since we are equally splitting available spectral resources between both transmissions.

Hereafter, we assume that spectral resources are shared and equally distributed between both transmissions, i.e. the maximal spectral efficiency for user 1 is then:

$$R_1(O_1, O_2) = \begin{cases} \frac{1}{2}\log_2(1+\gamma_1) & \text{if } O_2 = 2 \\ \frac{1}{2}\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right) & \text{if } O_2 \neq 2 \end{cases} \tag{5.5}$$

Note that the performance of the TDM/FDM technique, which allows to avoid strictly interference between users, allows for a minimal bound $r_\alpha = \frac{1}{2}$, for any value of α. That is the reason why, all 5 interference regimes performance illustrated in the W-shaped curve in Figure 5.1, are lower bounded by $r_\alpha > \frac{1}{2}$, otherwise it would mean that avoiding interference through orthogonalization would be provide the system with a better spectral efficiency after interference processing. Even though suboptimal, this approach is extremely simple and easy to implement in practice. This is the reason why, Abgrall [86] simplified the 3 in-between interference regimes, based on the theoretical studies from [83], into only one regime, based on orthogonalization.

5.2.4 Formulation of the Different Interference Regimes for Couples of Interferers

According to the previous sections, we can define 3 regimes for each pair source-destination, and their constraints on the spectral efficiencies of both users. From the point of view of user 1, this means that we have modified the W shaped curve into a discontinuous curve, represented in Figure 5.3, with only 3 regimes.

The presented analysis leads to 9 possible combinations of interference regimes $\mathcal{O} = (O_1, O_2)$, for our pair of users in the 2-GIC. We sum up in 5.2, the 9 possible combinations and the theoretical limits on the maximal spectral efficiencies for each user, so that the system can fall into each interference regime. Since some configurations are symmetric, only 6 regimes were listed in the following table.

Interference Regimes $\mathcal{O} = (O_1, O_2)$	Maximal spectral efficiency $R_1(O_1, O_2)$	Maximal spectral efficiency $R_2(O_1, O_2)$
$(1,1)$	$\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)$	$\log_2\left(1+\frac{\gamma_2}{1+\delta_2}\right)$
$(1,2)$	$\frac{1}{2}\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)+\frac{1}{2}\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)$	$\frac{1}{2}\log_2\left(1+\frac{\gamma_2}{1+\delta_2}\right)$
$(1,3)$	$\min\left[\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right), \log_2\left(1+\frac{\delta_2}{1+\gamma_2}\right)\right]$	$\log_2(1+\gamma_2)$
$(2,2)$	$\frac{1}{2}\log_2(1+\gamma_1)$	$\frac{1}{2}\log_2(1+\gamma_2)$
$(2,3)$	$\min\left[\frac{1}{2}\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right), \frac{1}{2}\log_2\left(1+\frac{\delta_2}{1+\gamma_2}\right)\right]$	$\log_2(1+\gamma_2)$
$(3,3)$	$\min\left[\log_2(1+\gamma_1), \log_2\left(1+\frac{\delta_2}{1+\gamma_2}\right)\right]$	$\min\left[\log_2(1+\gamma_2), \log_2\left(1+\frac{\delta_1}{1+\gamma_1}\right)\right]$

Table 5.2: Summary of regimes and their maximal spectral efficiencies

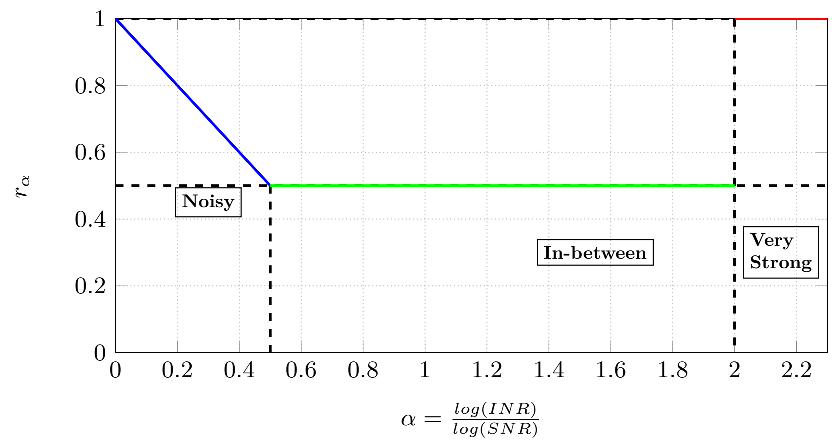

Figure 5.3: Generalized degrees of freedom, according to the α value, for the 3 regimes interference classification.

For any given channel configuration $\omega = (\gamma_1, \gamma_2, \delta_1, \delta_2)$, we define the total spectral efficiency $R(O_1, O_2)$ that the system can achieve with the interference regime $\mathcal{O} = (O_1, O_2)$, as the sum of the individual maximal spectral efficiencies, i.e. $R(O_1, O_2) = R_1(O_1, O_2) + R_2(O_1, O_2)$.

5.3 System Model and Optimization Problem Definition

5.3.1 Interference Classification: System Model

In this section, we investigate the best combination of interference regimes to be used, in any SNR/INR configuration of the 2-GIC, depicted in Figure 5.2. The system then simply consists of two User Equipments (UE) and two Access Points (AP), both indexed by $i \in \{1, 2\}$. For simplicity, each AP i is assigned to its UE i and competitively transmit, sharing the same geographical area and spectral resources. For simplification, in the following, the pair AP i - UE i will be called interferer i. Let us also define the channel matrix $\mathcal{H}$ as:

$$\mathcal{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \tag{5.6}$$

Where $\forall i, j \in \{1,2\}^2$, h_{ij} refers to the channel realization between AP i and UE j. Noise instances $(z_i)_{i\in\{1,2\}}$ are assumed to be i.i.d. random realizations of a white Gaussian noise process with zero mean and noise variance σ_n^2.

We voluntarily fix transmission powers and denote $\mathcal{P} = (p_1, p_2)$ the set of powers used for transmission, where p_i is the transmission power used at AP i. According to the previous notations, we can then define $\forall i \in \{1,2\}$, γ_i the signal to noise ratio (SNR) perceived by UE i and δ_i, the interference to noise ratio (INR) perceived by UE i, due to the interference generated by AP $j \in \{1,2\} \neq i$ as:

$$\gamma_i = \frac{p_i|h_{ii}|^2}{\sigma_n^2} \text{ and } \delta_i = \frac{p_j|h_{ji}|^2}{\sigma_n^2} \tag{5.7}$$

Finally, we denote, for each interferer i, O_i, the interference regime, i.e how the interference is treated by each interferer i. We assume that a central unit, aware of the 2-GIC configuration $\omega = (\gamma_1, \gamma_2, \delta_1, \delta_2)$ can the interference regime $\mathcal{O}$, which leads to individual maximal spectral efficiencies (R_1, R_2) for each user. The interference can be processed, according to the 3-regimes classification, introduced in [86, 87]:

$$O_i = \begin{cases} 1 & \text{if Noisy, as in Section 5.2.1} \\ 2 & \text{if Orthogonal Transmission as in Section 5.2.3} \\ 3 & \text{if SIC as in 5.2.2} \end{cases} \tag{5.8}$$

Details on each regime and their respective performance, in terms of limitations on the spectral efficiencies of each interferer, were provided in the Section 5.2.

5.3.2 Interference Classification: Optimization Problem

In this section, our objective is to define, for any given channel configuration $\omega = (\gamma_1, \gamma_2, \delta_1, \delta_2)$, the optimal interference regime $\mathcal{O}^* = (O_1^*, O_2^*)$, among the 9 possible interference regimes defined in Table 5.2, such that the total spectral efficiency of the system $R(O_1, O_2) = R_1(O_1, O_2) + R_2(O_1, O_2)$ is maximized. It is equivalent to the following optimization problem:

$$\begin{aligned} &\forall \omega,\ \mathcal{O}^* = (O_1^*, O_2^*) = \arg\max_{\mathcal{O}} \Big[R(O_1, O_2)\Big] \\ &\text{s.t. } R(O_1, O_2,) = R_1(O_1, O_2) + R_2(O_1, O_2) \end{aligned} \tag{5.9}$$

In our optimization problem, we assume that the central unit has perfect

CSI and may, at will and in coordination, change the interference regime $\mathcal{O}$, thus modifying the individual maximal spectral efficiency, as detailed in Table 5.2. The number of admissible interference regimes $\mathcal{O}$ is finite, which leads to a low-complexity problem that necessarily admits at least one optimal solution. The short-term configuration of transmission powers, SNRs and INRs remain unchanged, which does not lead to classical complications related to multiuser power control games. For example, the optimization process does not have to deal with *'ping pong effects'*. By *'ping pong effects'*, we refer to an optimization phenomenon, where an interferer may adapt its transmission power to the interference pattern he perceives, and as a consequence changes the interference pattern perceived by the other interferer. The second interferer then wishes to re-adapt its own transmission power to the new perceived interference, changing again the interference pattern perceived by the first user. Such an iterative process can eventually take a long time to converge, as detailed in Section 4.3.

Our optimization process leaves the short-term power configurations of each interferer and interference patterns unchanged and does not have to deal with such complications. Instead, the couple of interferers can deal with the way interference might be processed at each receiver side, i.e. select a combination of interference regimes, while aiming at maximizing the total spectral efficiency. After the conducted analysis, we obtain, for each possible regime (O_1, O_2), conditions that ω must verify so that the interference regime (O_1, O_2) is the best performing regime among all 9 possible ones. This leads to an interference classifier for our system of two interferers, that is detailed in the following sections.

5.3.3 Eliminating Outperformed Interference Regimes

Before starting the analysis, let us first define, the following operator, $\triangleright$, where $(O_1, O_2) \triangleright (O'_1, O'_2)$ means that the interference regime (O_1, O_2) outperforms (O'_1, O'_2), i.e. (O_1, O_2) offers a better maximal total spectral efficiency than (O'_1, O'_2), when the channel configuration is ω:

$$\begin{aligned} &(O_1, O_2) \triangleright (O'_1, O'_2) \Leftrightarrow R(O_1, O_2) \geq R(O'_1, O'_2) \\ &\text{with } \begin{cases} R(O_1, O_2) = R_1(O_1, O_2) + R_2(O_1, O_2) \\ R(O'_1, O'_2) = R_1(O'_1, O'_2) + R_2(O'_1, O'_2) \end{cases} \end{aligned} \tag{5.10}$$

Among the 9 possible regimes, only 4 regimes of interest can be dominant. For any given channel configuration ω, the 5 other regimes are always outperformed by at least one of the 4 regimes of interest. We can then define the 4 regimes of interest, thanks to Proposition 5.1

Proposition 5.1. *The following statements hold, for any given SNR/INR configuration* ω*:*

- $(2,1)$ *and* $(1,2)$ *are outperformed by either* $(2,2)$*,* $(1,1)$*,* $(3,1)$ *or* $(1,3)$*.*

- $(2,3)$ *and* $(3,2)$ *are respectively outperformed by* $(1,3)$ *and* $(3,1)$*.*

- $(2,2)$ *is outperformed by either* $(3,1)$ *or* $(1,3)$*.*

Proof. Refer to Appendix 8.4. □

Proposition 5.1 shows that the orthogonalization-based regimes are always outperformed by at least one of the non-orthogonalization-based regimes. As a consequence of the 3 previous propositions, we show that the study may be limited to only 4 regimes of interest: $(1,1)$, $(1,3)$, $(3,1)$ and $(3,3)$. This leads to a first interesting conclusion: when the system aims to maximize its total spectral efficiency, no user i will implement an orthogonalization-based strategy (i.e. a configuration $\mathcal{O}$ where $\exists i \in \{1,2\}, O_i = 2$). This first contribution shows that the system can deal with interference no matter what the channel configuration ω is, in a spectral efficient way, by either treating interference as noise or by limiting at least the spectral efficiency of one user so that the second user can decode interference and cancel it out via SIC-based techniques. The scenario, where interference is avoided by orthogonalizing transmissions, appears spectrally inefficient: such a result was expected, since it is well-known that orthogonalization and interference avoidance are highly inefficient, in terms of spectral efficiency. Now, we must define the SNR/INR ω regions of dominance for each regime of interest.

5.3.4 Best Performance Regions

In this section, we look forward to defining the regions of dominance of each regime of interest. It leads to an IC algorithm, with only two admissible regimes for each user i: noisy ($O_i = 1$) or SIC ($O_i = 3$). In the following propositions, we focus on defining the dominance criterion for any two regimes $(\mathcal{O}, \mathcal{O}')$, i.e. regimes of ω, where interference regime $\mathcal{O} \triangleright \mathcal{O}'$. The proofs of each proposition have been detailed in the Appendices Chapter, for readability's sake.

Proposition 5.2.

$$(1,1) \rhd (1,3) \Leftrightarrow \gamma_1 \geq \delta_2(1+\delta_1) \tag{5.11}$$

$$(1,1) \rhd (3,1) \Leftrightarrow \gamma_2 \geq \delta_1(1+\delta_2) \tag{5.12}$$

Proof. Refer to Appendix 8.5. □

Proposition 5.3. *A sufficient condition for* $(1,3) \rhd (3,3)$ *is* $\gamma_2 \geq \delta_1$*. A sufficient condition for* $(3,1) \rhd (3,3)$ *is* $\gamma_1 \geq \delta_2$*. Moreover, when* $\gamma_2 \leq \delta_1$ *and* $\gamma_1 \leq \delta_2$*, the two following statements hold:*

$$\left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2) \Leftrightarrow (3,3) \rhd (1,3)$$

$$\left(1+\frac{\delta_1}{1+\gamma_1}\right)(1+\delta_2) \geq (1+\gamma_1)(1+\gamma_2) \Leftrightarrow (3,3) \rhd (3,1)$$

Proof. Refer to Appendix 8.6. □

Based on these two propositions, we can define the regions on ω, in which (1,1) and (3,3) are the best performing regimes, as well as regions where none of these two regimes are best performing regimes, according to both Propositions 5.4 and 5.5.

Proposition 5.4. $(1,1)$ *is the best interference regime if and only if* $(\gamma_1, \gamma_2, \delta_1, \delta_2)$ *verify the two following statements:*

$$\begin{cases} \gamma_1 \geq (1+\delta_1)\delta_2 \\ \gamma_2 \geq (1+\delta_2)\delta_1 \end{cases} \tag{5.13}$$

Proof. Refer to Appendix 8.7. □

Proposition 5.5. $(3,3)$ *is the best interference regime if and only if* $(\gamma_1, \gamma_2, \delta_1, \delta_2)$ *verify the four following statements:*

$$\begin{cases} \gamma_1 \leq \delta_2 \\ \gamma_2 \leq \delta_1 \\ (1+\gamma_1)(1+\gamma_2) \leq (1+\delta_1)\left(1+\frac{\delta_2}{1+\gamma_2}\right) \\ (1+\gamma_1)(1+\gamma_2) \leq (1+\delta_2)\left(1+\frac{\delta_1}{1+\gamma_1}\right) \end{cases} \tag{5.14}$$

Proof. Refer to Appendix 8.8. □

With the two previous propositions, we have defined the regions of dominance for $(1,1)$ and $(3,3)$. When $(\gamma_1,\gamma_2,\delta_1,\delta_2)$ does not verify any of the two previous propositions, then the best interference regime is either $(1,3)$ or $(3,1)$. The decision on whether $(1,3)$ or $(3,1)$ is the best regime is given by Proposition 5.6.

Proposition 5.6. *When $(\gamma_1,\gamma_2,\delta_1,\delta_2)$ does not satisfy the conditions of either Proposition 5.4 or Proposition 5.5, two cases can be considered:*

- *if $\gamma_1 < \delta_2$ and $\gamma_2 < \delta_1$, then $(1,3) \triangleright (3,1) \Leftrightarrow (1+\gamma_1+\delta_1)\gamma_2\delta_2 \geq (1+\gamma_2+\delta_2)\gamma_1\delta_1$.*
- *else if $\gamma_1 \geq \delta_2$ or $\gamma_2 \geq \delta_1$, then $(1,3) \triangleright (3,1) \Leftrightarrow \gamma_2 \geq \gamma_1+(\delta_1-\delta_2)$.*

Proof. Refer to Appendix 8.9. □

5.3.5 The Proposed Two-Regimes Interference Classification

Based on the previous propositions, we can observe that each interferer i only implements two interference regimes: $O_i = 1$(noisy) or $O_i = 3$(SIC). We can then define in Proposition 5.7, a '2-Regimes Interference Classification' Algorithm, when two interferers are coupled together. Our low-complexity classification algorithm is cascaded: it first checks if the SNR/INR configuration ω falls into the best performance regions related to $(1,1)$ and $(3,3)$. If it turns out that ω does not satisfy the conditions of either Proposition 5.4 or Proposition 5.5, then the algorithm checks which one performs the best between $(1,3)$ and $(3,1)$, according to Proposition 5.6.

Proposition 5.7. *For any given ω,*

- $(1,1)$ *is the best interference regime if and only if $\gamma_1 \geq (1+\delta_1)\delta_2$ and $\gamma_2 \geq (1+\delta_2)\delta_1$.*
- $(3,3)$ *is the best interference regime if and only if $(\gamma_1,\gamma_2,\delta_1,\delta_2)$ verify the four following statements:*

$$\begin{cases} \gamma_1 \leq \delta_2 \\ \gamma_2 \leq \delta_1 \\ (1+\gamma_1)(1+\gamma_2) \leq (1+\delta_1)\left(1+\frac{\delta_2}{1+\gamma_2}\right) \\ (1+\gamma_1)(1+\gamma_2) \leq (1+\delta_2)\left(1+\frac{\delta_1}{1+\gamma_1}\right) \end{cases} \tag{5.15}$$

- *When* $(\gamma_1, \gamma_2, \delta_1, \delta_2)$ *does not satisfy the conditions any of the two previous propositions,* $(1, 3)$ *is the best performing regime if it satisfies any of the two following propositions:*

  i) if $\gamma_1 < \delta_2$ *and* $\gamma_2 < \delta_1$*, and* $(1+\gamma_1\delta_1)\gamma_2\delta_2 \geq (1+\gamma_2+\delta_2)\gamma_1\delta_1$.

  ii) else if $(\gamma_1 \geq \delta_2$ *or* $\gamma_2 \geq \delta_1)$ *and* $\gamma_2 \geq \gamma_1+(\delta_1-\delta_2)$.

  Otherwise, $(3, 1)$ *is the best performing regime.*

We have then defined a two-regimes classification algorithm, that returns the optimal interference regimes to be used in any configuration ω of a 2-GIC. The optimal regime consisted of the regime which maximized the total spectral efficiency, after interference processing at each receiver side. In the following section, we propose to exploit this 2-regimes classification, into a matching problem, which consists of 2 APs, with multiple UEs assigned to each AP. The objective consists of forming pairs of interferers, with one interferer from each AP, that transmit using the same spectral resources, interfere, but are able to adapt their interference regimes and spectral efficiencies, according to our '2-Regimes Interference Classification'.

5.4 Matching Interferers with Interference Classification: a First Scenario with $M = 2$ APs and Coalitions

5.4.1 System Model Update

In this section, and by analogy with the system model detailed in Section 5.3, we define a matching problem where $M = 2$ coalitions of N interferers, each coalition being assigned to one AP, have to be matched together. We denote UE (k, i), where $k \in \{1, 2\}$ and $i \in \mathcal{N} = \{1, ..., N\}$, the UE i belonging to the users coalition k. We consider a downlink transmission from each AP k to all its assigned UEs $(k, i), i \in \mathcal{N}$. In the following, 'interferer (k, i)' refers to the combination AP k - UE (k, i). We now denote h_i^{kl} the channel between AP k and UE (l, i). Transmission powers are fixed: p_i^k denotes the transmission power from AP k to its assigned UE (k, i). We assume that the spectral resources available in the system are split in N equal parts $\{S_1, S_2, ..., S_N\}$. The general formulation of the optimization problem is presented in Section 5.5.1.

If interferers $(1, i_1)$ and $(2, i_2)$ transmit using the same spectral resource S_k, they suffer from interference: as before in Section 5.3, $\forall (i_1, i_2) \in \mathcal{N}^2$, a 2-GIC is considered for the group of interferers $[(1, i_1), (2, i_2)]$. Based on the previous definitions, we can also re-define $\gamma(k, i)$, the SNR perceived by UE (k, i), due to an incoming transmission from its associated AP k. In a similar way, $\delta(k, l, i, j)$ denotes the INR perceived by UE (k, i) due to interference, related to the incoming transmission from AP l to its UE (l, j). Note that the INR criterion only makes sense, when $l \neq k$, as:

$$\gamma(k,i) = \frac{p_i^k |h_i^{kk}|^2}{\sigma_n^2} \text{ and } \delta(k,l,i,j) = \frac{p_j^l |h_i^{lk}|^2}{\sigma_n^2} \tag{5.16}$$

Where we denote σ_n^2 the noise variance. Finally, we denote $\omega(i_1, i_2)$ the set of SNRs/INRs related to the group of interferers $[(1, i_1), (2, i_2)]$, i.e.:

$$\omega(i_1, i_2) = (\gamma(1, i_1), \gamma(2, i_2), \delta(1, 2, i_1, i_2), \delta(2, 1, i_2, i_1)) \tag{5.17}$$

5.4.2 Reformulating the Optimization Problem

In this section, we reformulate the optimization problem, to take into account the multiple users assigned to each AP. Our objective is to the optimal matching m^* and interference regimes $\bar{\mathcal{O}}^*$), where

- m is a bijective matching between the N users of each coalition, where $\forall (i_1, i_2) \in \mathcal{N}^2$, $m(i_1, i_2) = 1$ if the interferers $(1, i_1)$ and $(2, i_2)$ are matched together and share the same spectral resources. Otherwise, $m(i_1, i_2) = 0$. An example of a valid bijective matching is given in Figure 5.4. The bijective property allows for no interferer to be left unassigned to another interferer and ensures that each group of interferers affected to each spectral resource S_i ($i \in \mathcal{N}$) consists of exactly one user from each coalition.

- $\bar{\mathcal{O}}$ corresponds to the interference regimes for each couple (i_1, i_2) defined by m:

$$\bar{\mathcal{O}} = \left\{ \mathcal{O}(i_1, i_2) \mid (i_1, i_2) \in \mathcal{N}^2, m(i_1, i_2) = 1 \right\} \tag{5.18}$$

$$\mathcal{O}(i_1, i_2) = (O_1(i_1, i_2), O_2(i_2, i_1)) \tag{5.19}$$

such that the total spectral efficiency of the system $U(m, \bar{\mathcal{O}})$, defined hereafter, is maximized.

$$U = \sum_{i_1=1}^{N} \sum_{i_2=1}^{N} m(i_1, i_2) R\left(i_1, i_2, \mathcal{O}(i_1, i_2)\right) \tag{5.20}$$

We also denote $R(i_1, i_2, \mathcal{O}(i_1, i_2))$ as the total spectral efficiency for the couple of interferers $[(1, i_1), (2, i_2)]$, when the interference regime is $\mathcal{O}(i_1, i_2)$ and the 2-GIC configuration is $\omega(i_1, i_2)$.

For any two given users i_1 in coalition 1 and i_2 in coalition 2, the admissible interference regimes for $\mathcal{O}(i_1, i_2)$ are the 4 regimes of interest that were defined in Section 5.3.3, i.e: $(1, 1)$, $(1, 3)$, $(3, 1)$ and $(3, 3)$. The best interference regime $\mathcal{O}^*(i_1, i_2)$ among the 4 regimes of interest is given by our 2-Regimes Interference Classifier, defined previously in Proposition 5.7, assuming $\omega(i_1, i_2)$ plays the exact same role as ω, for a set of two interferers $[(1, i_1), (2, i_2)]$. Our objective is then to solve the following optimization problem:

$$(m^*, \mathcal{O}^*) = \arg \max_{(m, \mathcal{O})} \left[U(m, \mathcal{O})\right] \tag{5.21}$$

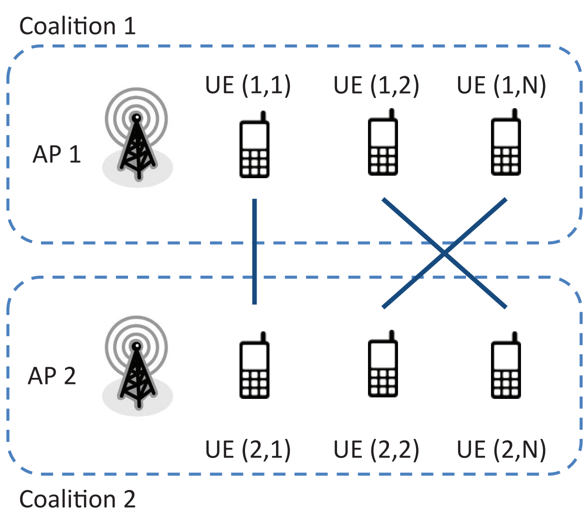

Figure 5.4: A possible matching with one interferer from each coalition ($M = 2$, $N = 3$). Two coalitions of 3 UEs assigned to each AP have been represented.

Reusing our previous classifier, we are able to solve half of the optimization problem. In fact, for any two given users i_1 in coalition 1 and i_2 in coalition 2, we can define the best interference regime $\mathcal{O}^*(i_1, i_2)$ and its total spectral efficiency performance $C(i_1, i_2) = R(i_1, i_2, \mathcal{O}^*(i_1, i_2))$. Our objective, is now to find, among the $N!$ bijective matchings m, the one that maximizes the total spectral efficiency of the system $U(m, \mathcal{O})$, which is now defined as $U'(m)$, i.e.:

$$(m^*) = \arg\max_{(m)} [U'(m)] \tag{5.22}$$

$$U'(m) = \sum_{i_1=1}^{N} \sum_{i_2=1}^{N} m(i_1, i_2) C(i_1, i_2) \tag{5.23}$$

Among the two coalitions of N interferers, we seek the optimal way to form N couples of interferers, with one interferer from each coalition, assuming interferers implement IC, and do so that the total spectral efficiency of the system is maximized. From one interferer point of view, this means that we are looking for a *'friendly' interferer* in the opposite coalition, whose interference is either:

- weak enough, so that reliable transmission can occur by treating interference as an additive source of noise,
- or strong enough, so that interference can be decoded and canceled out, via SIC-based techniques.

5.4.3 A 2-Dimensional Assignment Problem

In the previous section, we have defined a performance matrix C, whose general term $C(i_1, i_2)$ is the combined spectral efficiency that interferer $(1, i_1)$ and interferer $(2, i_2)$ could enjoy if they were paired together into a group S_i ($i \in \mathcal{N}$).

From the matrix point-of-view, the second step of the original optimization problem, defined in Equation (5.22), is then strictly equivalent to a planar assignment problem [184], applied on matrix C. Indeed, finding the optimal bijective matching m^* is then strictly equivalent to finding the combinations of N terms in matrix C, such that:

- there is one and only one selected term on each row and column, this condition guarantees that the assignment m is bijective;
- and the sum of the N terms is maximal, which is strictly equivalent to the assignment m maximizing $U'(m)$.

Such an assignment problem can be solved, by a low-complexity optimal algorithm: the Kuhn-Munkres algorithm [96, 98]. Using simple operations on matrix

C, the Kuhn-Munkres algorithm returns, in polynomial computation time, the maximal bijective combination m^* of N terms in matrix C.

5.4.4 Numerical Results and Performance Improvements ($M = 2$ scenario)

In this section, we demonstrate that our classification and matching algorithms, both offer significant gains, in terms of total spectral efficiency, compared to 4 reference Interference Management Strategies (IMS). The IMS under study are listed hereafter. Note that the first two IMS may be considered as state of the art, whereas the next 3 IMS are used to show the benefits offered by either the IC and/or the matching of interferers.

1. **IMS 1:** $(2,2)$ **only**: this scenario corresponds to the case where the only way to process interference is orthogonalization. Interference does not apply, since interferers do not share spectral resources anymore. Every matching of interferers returns the same result. The total spectral efficiency performance of the system is then given by U_{22}:

$$U_{22} = \sum_{j=1}^{2} \sum_{j=1}^{N} \frac{1}{2} \log_2 (1+\gamma(j,i)) \tag{5.24}$$

2. **IMS 2:** $(1,1)$ **& Random Matching**: in this scenario, the only way to process interference is treating it as additive noise, i.e. for any group of interferers, the interference regime is $(1,1)$. We also assume that the system can not perform any smart matching, and couples UE $(1,i)$ to UE $(2,i)$, into group S_i $(i \in \mathcal{N})$. The total spectral efficiency performance, denoted U_{11r} is then given by:

$$U_{11r} = \sum_{j=1}^{2} \sum_{i=1}^{N} \log_2 \left(1+\frac{\gamma(j,i)}{\mathcal{I}(j,i)}\right) \tag{5.25}$$

$$\mathcal{I}(j,i) = 1+\sum_{\substack{k=1 \\ k\neq j}}^{2} \delta(j,k,i,i) \tag{5.26}$$

3. **IMS 3:** $(1,1)$ **& Best Matching**: as in 2), the only way to process interference corresponds to the noisy regime. But it differs by the fact that the system is given the possibility to define the most appropriate

matching. To do so, we define the following performance matrix C', whose general term is given by:

$$C'(i_1, i_2) = \sum_{j=1}^{2} \log_2 \left(1 + \frac{\gamma(j, i_j)}{\mathcal{I}(j, i_j)} \right) \tag{5.27}$$

$$\mathcal{I}(j, i) = 1 + \sum_{\substack{k=1 \\ k \neq j}}^{2} \delta(j, k, i, i) \tag{5.28}$$

Applying the Kuhn-Munkres algorithm to C' returns the best performing matching m'^{*}, from which we define the total performance U_{11bm} as:

$$U_{11bm} = \sum_{i_1=1}^{N} \sum_{i_2=1}^{N} m'^{*}(i_1, i_2).C'(i_1, i_2) \tag{5.29}$$

4. **IMS 4: Best Regime & Random Matching**: in this scenario, we assume that the system can not perform any smart matching, and couples interferer $(1, i)$ to interferer $(2, i)$, into group S_i $(i \in \mathcal{N})$. The system can however define the most appropriate regime $\mathcal{O}(i, i)$, among the 4 regimes for interest for every pair of interferers $[(1, i), (2, i)]$. The total performance of the system in such a scenario is U_{BRr}, which is simply defined as:

$$U_{BRr} = \sum_{i=1}^{N} C(i, i) \tag{5.30}$$

5. **IMS 5: Best Regime & Matching scenario**: the optimal scenario under investigation. We denote U_{Opt} the total spectral performance we obtain by solving our previous optimization problem.

In the numerical simulations, we have considered a system with two APs, within a distance of d_{AP}. The N users of each coalition are uniformly distributed in the coverage area of each AP R_{AP}. In the following, we denote $d(i, j, k)$ the distance between AP i and UE (j, k). The channels h_i^{lk} include the antenna gain G, the path loss $L(d(i, l, k))$ and the shadowing ξ. All parameters are summarized in Table 5.3, and are based on [185].

Figure 5.5 shows one realization of the network deployment under investigation.

In this configuration, we run Monte-Carlo simulations, with $N_{MC} = 1000$

Parameter	Value
Distance between AP to AP d_{AP}	1km
Coverage Area R_{AP}	Users are unif. dist. ,s.t. dist. AP-UE $\in [r_{min}, r_{max}]$
$[r_{min}, r_{max}]$	$[35m, 750m]$
Transmission power p_k^i	Unif. Dist. between 20 and 46 dBm
Channels h_i^{lk}	$h_i^{lk} = \frac{G}{L(d(i,l,k)).\xi}$
Antenna Gain G	10 dBi
Path Loss $L(d(i,l,k))$, [d in km]	$L = 131.1{+}42.8log_{10}(d(i,l,k))$
Shadowing ξ	Log-normal, $\sigma_{SH} = 10$ dB
Noise power σ_n	-104 dBm
Number of UEs per coalition N	20

Table 5.3: Simulations Parameters

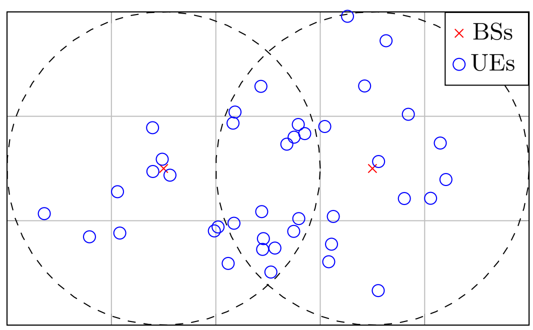

Figure 5.5: One instance of the network deployment under investigation - $M = 2$ APs and $N = 20$ UEs/AP.

independent iterations and have compared the mean performances of each IMS, i.e. the averaged values of U_{22}, U_{11r}, U_{11bm}, U_{BRr} and U_{Opt}. Figure 5.6 represents the histogram plot of the performance realizations of each scenario. Average total spectral efficiency values obtained for each scenario have also been displayed.

It immediately appears, as expected, that the orthogonalization strategy, namely the IMS 1, is highly inefficient, compared to the 4 other IMS. Secondly, it appears that allowing the system to select the best regime among the 4 regimes of interest, allows an enhancement of the average performance: Monte-Carlo simulations show that the average performance improvement is 5.25%: this is obtained by comparing both IMS 2 and IMS 4. The same way, allowing the system to smartly couple interferers in a constrained (1,1) scenario (i.e. IMS 3)

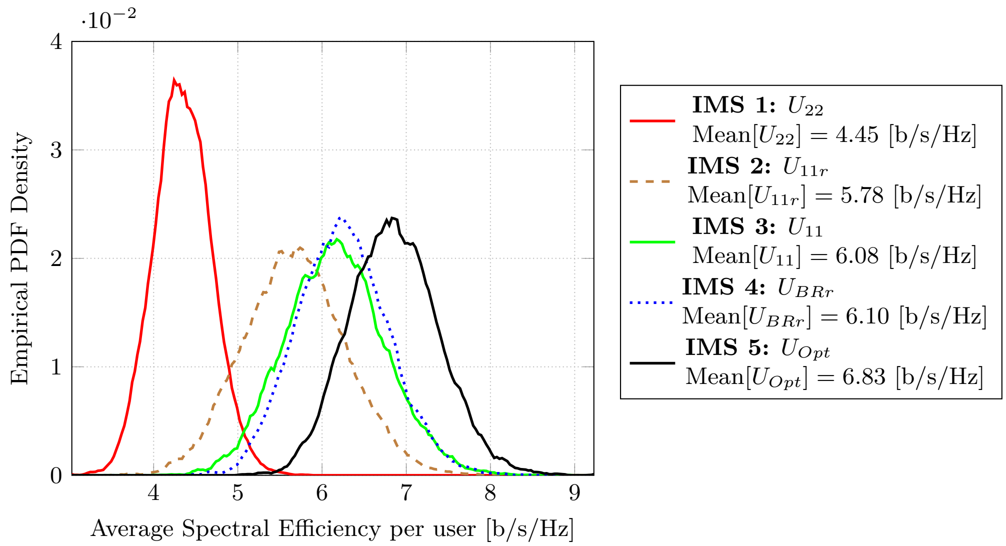

Figure 5.6: Histogram of the total spectral efficiencies of each scenario under study, over $N_{MC} = 1000$ independent Monte-Carlo simulations.

also allows for an enhancement of the average spectral efficiency, compared to IMS 2: the performance improvement offered by the matching reaches 5.09%. In the end, we consider the IMS 5 combining both improvements: smartly selecting the best interference regime and smartly matching interferers grants the system an average performance improvement of 15.37% compared to the IMS 2.

5.5 Matching Interferers with Interference Classification: Extension to $M > 2$ APs and Coalitions

In this section, we investigate the extension of the matching problem detailed in Section 5.4, to the case where the number of different APs and coalitions are $M > 2$. The investigation becomes more complex, for two reasons, explicitly detailed in Section 5.5.2.

5.5.1 System Model and the Optimization Problem Update

In this section, the system now consists of a set of $M > 2$ Access Points and $M > 2$ coalitions of N UEs assigned to each AP, sharing the same geographical area. If interferers $[(1, i_1), ..., (M, i_M)]$ share the same spectral resources, they suffer from interference: $\forall(i_1, ..., i_M) \in \mathcal{N}^M$, a M-GIC is considered for the group of interferers $[(1, i_1), ..., (M, i_M)]$, as depicted on Figure 5.7. The notations for the powers, channels, SNRs and INRs from Section 5.4 still hold. However, we now denote $\omega(i_1, i_2, ...i_M)$ the set of SNRs/INRs related to a group of interferers $[(1, i_1), (2, i_2), ..., (M, i_M)]$, where:

$$\omega(i_1, i_2, ...i_M) = \Big[\Gamma(i_1, i_2, ...i_M), \Delta(i_1, i_2, ...i_M)\Big] \tag{5.31}$$

$$\Gamma(i_1, i_2, ...i_M) = \Big[\gamma(1, i_1), ..., \gamma(M, i_M)\Big] \tag{5.32}$$

$$\Delta(i_1, i_2, ...i_M) = \Big[\delta(j, k, i_j, i_k) \mid j, k \in \mathcal{M} \text{ and } j \neq k\Big] \tag{5.33}$$

Where $\mathcal{M} = \{1, ..., M\}$.

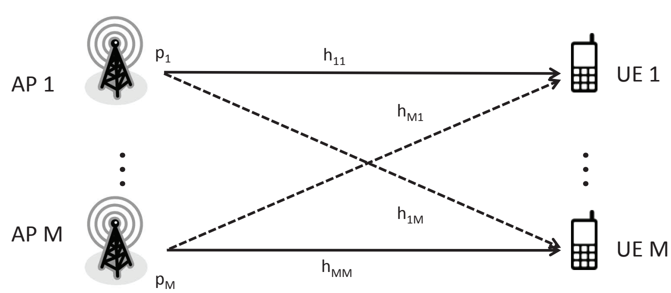

Figure 5.7: The M-users Gaussian interference channel, with $M > 2$.

5.5.2 Limitations on Interference Classification and Interferers Matching in the $M > 2$ Scenario

When the number of interferers coupled together was $M = 2$, we can define, according to our previous study in Section 5.3, the best way to process the interference, so that the total spectral efficiency for the couple of interferers is maximized. However, when the number of interferers coupled together becomes larger than 2 (i.e. $M > 2$), defining the best interference regime becomes a lot more complex. Several papers, as [88], have tried to tackle the problem of both defining the best way to process interference in a multiple interferers scenario and estimating the system inherent spectral efficiency. However, when IC is considered, Group SIC, iterative k-SIC or k-Joint Decoding approaches come into play and complexify the analysis, as pointed out in [90], leading to multiple new regimes. Investigating these regimes requires to take into account the fact that decoding interference signals at each receiver is affected by the joint effect of interference, rather than each interfering signal. Therefore, it appears that it is better to consider directly the effect of the combined interference signal.

From one user point of view, implementing iterative SIC requires to define the set of interferers to be decoded as well as the order for decoding those interferers, which is something that did not have to be done in the $M = 2$ scenario. When M grows large, the number of possible interference regimes becomes extremely large as well, which rapidly renders impossible a detailed study of every single interference regime, as we have done in the 2-GIC.

Recently, interference alignment techniques have also been proposed that are based on this principle. The use of these techniques leads to achievable spectral efficiencies that, in some cases, can be as good as those over the 2-GIC [92, 93]. However, these techniques remain theoretical and does not suit well practical implementation [91]. For this reason and for the sake of simplicity, we assume, in the following, that all interferers process the incoming interference as additive noise and focus only on finding the best matching. For a given group of interferers $[(1, i_1), ..., (M, i_M)]$, we can then define the general term of what is now a N^M tensor C, $C(i_1, ..., i_M)$, as the sum of all individual spectral efficiencies $\tilde{R}(k)$ of each interferer (k, i_k), obtained by treating all interference as noise, i.e.:

$$C(i_1, ..., i_M) = \sum_{k=1}^{M} \tilde{R}(k) \tag{5.34}$$

$$\text{where, } \tilde{R}(k) = \log_2 \left(1 + \frac{\gamma(k, i_k)}{1 + \sum_{\substack{j=1 \\ j \neq k}}^{M}} \delta(k, j, i_k, i_j) \right) \tag{5.35}$$

Let us also define the following matching parameter m, as a N^M tensor, whose general term $m(i_1, i_2, ...i_M)$ is defined $\forall(i_1, i_2, ..., i_M) \in \mathcal{N}^M$, as:

$$m(i_1, i_2, ..., i_M) = \begin{cases} 1 & \text{if interferers } [(1, i_1), ...(M, i_M)] \text{are matched together} \\ 0 & \text{else} \end{cases} \tag{5.36}$$

Our objective in this paper, is to define the optimal bijective matching m^* such that U, defined as follows, is maximized:

$$U(m) = \sum_{i_1=1}^{N} ... \sum_{i_M=1}^{N} m(i_1, ..., i_M) C(i_1, ..., i_M) \tag{5.37}$$

The matching also has to be bijective, i.e the matching must guarantee that there is exactly one user from each coalition assigned to each spectral resource S_i ($i \in \mathcal{N}$). This means that the following constraints have to be verified, $\forall(i_1, i_2, ...i_M) \in \mathcal{N}^M$:

$$\begin{cases} \sum_{j_2=1}^{N} ... \sum_{j_M=1}^{N} m(i_1, j_2, ..., j_M) = 1 \\ \vdots \\ \sum_{j_2=1}^{N} ... \sum_{j_{k-1}=1}^{N} \sum_{j_{k+1}=1}^{N} ... \sum_{j_M=1}^{N} m(j_1, ..., j_{k-1}, i_k, j_{k+1}, ..., j_M) = 1 \\ \vdots \\ \sum_{j_1=1}^{N} ... \sum_{j_{M-1}=1}^{N} m(k_1, ..., k_{M-1}, i_M) = 1 \end{cases} \tag{5.38}$$

When $M > 2$, we have then defined a planar Multidimensional Assignment Problem (MAP), which is known to be NP-Hard [176]: the number of combinations that a brute-force algorithm needs to try out before finding the maximum matching is $(N!)^{(M-1)}$ and at the bast of our knowledge, there does not exist an algorithm that is both optimal and running in a polynomial computation time. It does appear though that suboptimal approaches based on the Kuhn-Munkres algorithm can be designed: in [186], the authors propose that a method extending the Kuhn-Munkres algorithm to the case $M = 3$, and other variants have been described by Kuhn in [97]. Throughout this section, we have implemented two suboptimal algorithms for the $M > 2$ case:

- the first one is an iterative layered low-complexity approach to the MAP, that we proposed and described in Section 5.5.3.

- the second is a memetic algorithm, which is able to approach the optimal solution of the assignment problem in an acceptable computation time. This algorithm is extensively detailed in [99].

It is impossible to tell how close the performance of the matching returned by the memetic algorithm is to the optimal matching, since no optimal algorithm is able to run in acceptable computation times, for large dimension systems. For this reason, we assume that the spectral efficiency performance of the suboptimal matching given by the memetic algorithm is the optimal spectral efficiency performance one could access realistically, and will now be considered as the optimum equivalent. A Integer Linear Programming (ILP) approach to the MAP could have also been considered, who could be solved optimally by branch-and-bound algorithms, in acceptable times, as long as the system dimensions N and M are low [177]. However, in this paper, we preferred the memetic approach, for complexity and computation speed reasons. Note that the performances of both the ILP algorithms and memetic approaches are close enough and often identical, in small dimension systems, to be considered equivalent.

5.5.3 Proposed Iterative Suboptimal MAP Algorithm

In this section, we propose a suboptimal procedure for solving the MAP in this paper. This second algorithm re-employs the low-complexity optimal Kuhn-Munkres algorithm, described in Section 5.4.3, in a layered iterative way, as suggested in [187]. Our approach is iterative in the sense that the Kuhn-Munkres algorithm is used $(M-1)$ times, to update the groups of users $(S_i)_{i\in\mathcal{N}}$. However suboptimal, our algorithm returns a matching m^* with notable performance improvements compared to a 'random' group formation, with low complexity and fast computation. In this section, we propose a suboptimal procedure for solving the MAP in this paper. This second algorithm re-employs the low-complexity optimal Kuhn-Munkres algorithm, described in Section 5.4.3, in a layered iterative way, as suggested in [187]. Our approach is iterative in the sense that the Kuhn-Munkres algorithm is used $(M-1)$ times, to update the groups of users $(S_i)_{i\in\mathcal{N}}$. However suboptimal, our algorithm returns a matching m^* with notable performance improvement compared to a 'random' group formation, with low complexity and fast computation.

In this section, the groups and spectral resources are both combined into the notation S_i, meaning that users in group S_i will use the spectral resource S_i. The group S_i consists of a vector of M elements, where $\forall(i_1, ...i_M) \in \mathcal{N}^M$, $S_i = (i_1, ...i_M)$ means that interferers $[(1, i_1), ..., (M, i_M)]$ belong to the same group and share spectral resource S_i. Figure 5.8 provides an illustrative diagram of the proposed algorithm, which is extensively detailed hereafter.

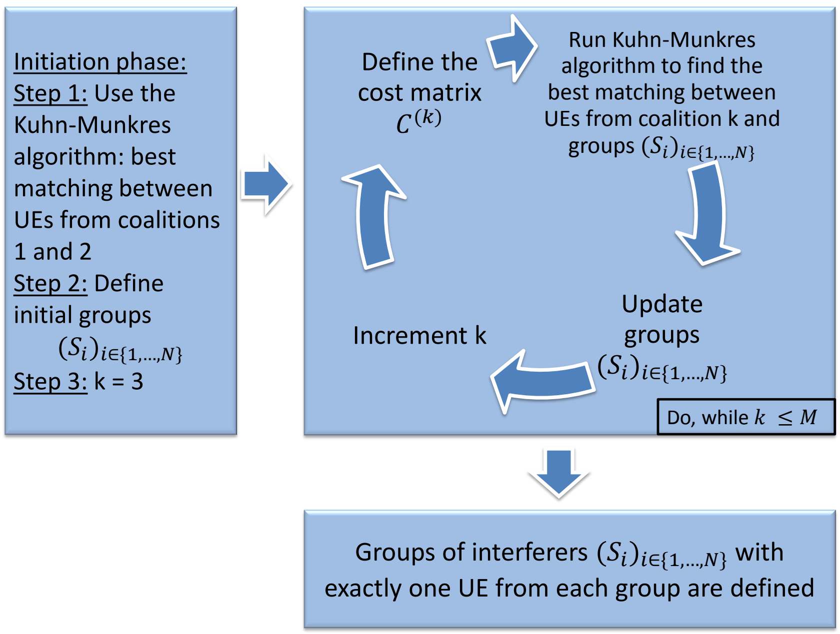

Figure 5.8: An illustration of the proposed iterative Kuhn-Munkres algorithm.

First, in the initiation phase, the proposed algorithm runs a Kuhn-Munkres, to define the best matching between the first two coalitions of interferers, just as we did in Section 5.4.3. Based on this first matching, groups $(S_i)_{i\in\mathcal{N}}$ are initialized, i.e. the first two elements of each vector (S_i) have values in $\mathcal{N}$, while the $(M-2)$ remaining entries are zeroes. During the iterative phase, illustrated in the do while block, the algorithm uses the Kuhn-Munkres algorithm again to realize a matching between the current groups of $(k-1)$ interferers and interferers from coalition k. At each step, the cost matrix $C^{(k)}$ considered, whose general term $C^{(k)}(i, j)$ corresponds to the total spectral efficiency that would be offered to the k-th interferers group, if it was formed by the union of the $(k-1)$ interferers currently assigned to group S_i and interferer (k, j). It can be decomposed into a sum of two elements:

- the spectral efficiency offered to interferer (k, j) if it joins in group S_i, i.e. the spectral efficiency offered to UE (k, j) assuming it suffers from interference due to interferers in group S_i
- the total spectral efficiency of the $(k-1)$ interferers from group S_i, assuming they now suffer from interference due to interferer (k, j)

The general term is then defined as:

$$\begin{aligned} C^{(k)}(i,j) = \quad & \log_2\left(1+\frac{\gamma(k,j)}{1+\sum_{l=1}^{k-1}\delta(k,l,j,S_i(l))}\right) \\ & +\sum_{l=1}^{k-1}\log_2\left(1+\frac{\gamma(l,S_i(l))}{1+\delta(l,k,S_i(l),j)+\sum_{\substack{m=1\\m\neq l}}^{k-1}\delta(l,m,S_i(l),S_i(m))}\right) \end{aligned} \tag{5.39}$$

Once the optimal matching between existing groups S_i and interferers from coalition k is found, the algorithm updates the groups S_i, with the new interferers assigned by the Kuhn-Munkres algorithm to each group S_i. As a consequence, the iterative phase goes on $(M-2)$ times in a row, so that the algorithm can process the M coalitions of interferers and include all interferers in one group of interferers.

5.5.4 Numerical Results and Performance Improvements ($M > 2$ scenario)

In this section, we use the same simulation parameters as in Section 5.4.4, which are listed in Table 5.3 . However, we consider a network consisting of $M = 5$ APs. For simplicity, we consider coalitions of $N = 7$ interferers per AP. One instance of the network deployment is shown in Figure 5.9.

In the numerical simulations results presented hereafter, we have considered 4 IMS for performance comparison.

1. **IMS 1:** $(2, 2)$ **only**: this scenario corresponds to the case where the only way to process interference is orthogonalization. Interference does not apply, since interferers do not share spectral resources anymore. Every matching of interferers returns the same result. The total spectral efficiency performance of the system is then given by U_{22}:

$$U_{22} = \sum_{j=1}^{M}\sum_{j=1}^{N}\frac{1}{2}\log_2\left(1+\gamma(j,i)\right) \tag{5.40}$$

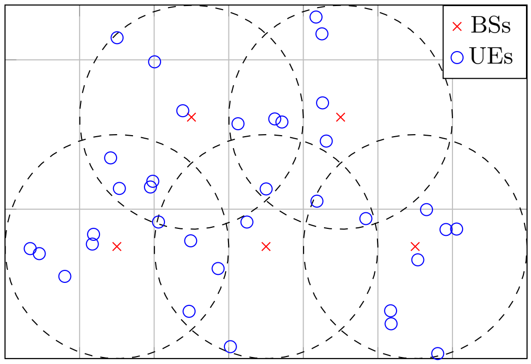

Figure 5.9: One instance of network deployment with $M = 5$ APs. For simplicity, we consider coalitions of $N = 7$ interferers per AP.

2. **IMS 2:** $(1, 1)$ **& Random Matching**: in this scenario, the only way to process interference is treating it as additive noise, i.e. for any group of users, the interference regime is $(1, 1)$. We also assume that the system can not perform any smart matching, and couples UEs $[(1, i), ...(M, i)]$ together, into group S_i ($i \in \mathcal{N}$). The total spectral efficiency performance, denoted U_{rand} is then given by:

$$U_{rand} = \sum_{j=1}^{M} \sum_{i=1}^{N} \log_2 \left(1 + \frac{\gamma(j, i)}{\mathcal{I}(j, i)} \right) \tag{5.41}$$

$$\mathcal{I}(j, i) = 1 + \sum_{\substack{k=1 \\ k \neq j}}^{M} \delta(j, k, i, i) \tag{5.42}$$

3. **IMS 3: Memetic Matching**: in this case, the system has no other choice but to treat interference as noise but can define the best matching of interferers, using the memetic algorithm we mentioned in the previous section. The performance of such a matching is noted U_{MM}

4. **IMS 4: Iterative Kuhn-Munkres Matching**: in this case, the system has no other choice but to treat interference as noise but can define the best matching of interferers, using the iterative Kuhn-Munkres algorithm we mentioned in the previous section. The performance of such a matching is noted U_{IKM}

In this configuration, we have run a Monte-Carlo process, with $N_{MC} = 1000$

iterations and have compared the mean performances of each scenario under study. The performance results for each scenario are presented in Figures 5.10 and 5.11.

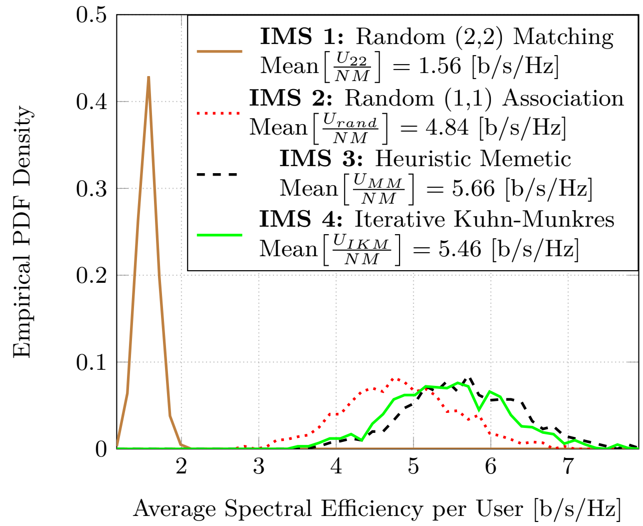

Figure 5.10: Zoom on the histogram of the total spectral efficiencies of each scenario under study, over $N_{MC} = 1000$ independent Monte-Carlo simulations.

Figure 5.10 represents the histogram of the $N_{MC} = 1000$ realizations of each scenario of interest. It appears, as expected, that a full orthogonalization scheme is highly inefficient, compared to any other scenario where interference might be treated as noise. Figure 5.11 focuses on the last 3 IMS. On average, it appears that the best matching, returned by the memetic algorithm, offers a significant improvement of the average total spectral efficiency performance of the system, compared to the IMS 2. This improvement is estimated around 17.1%. The algorithm we designed (IMS 4) is clearly suboptimal, but manages to offer an improvement of 13% in terms of averaged total spectral efficiency, compared to the random association scenario. As a consequence, it appears that looking for a smart coupling of interferers yields notable average performance improvements compared to scenarios of random association.

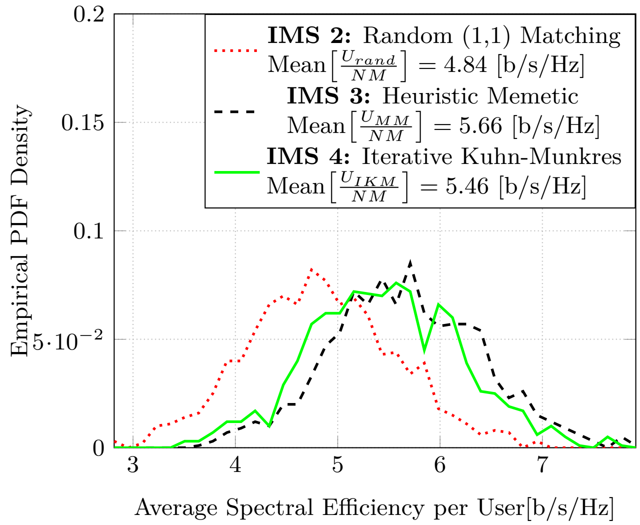

Figure 5.11: Zoom on the histogram of the total spectral efficiencies of each scenario under study, over $N_{MC} = 1000$ independent Monte-Carlo simulations.

5.6 Conclusions and Limits

In this chapter, we investigated a RRM technique, that could exploit the recent advances in terms of IC from Abgrall [86, 87]. The objective of the conducted optimization was to adapt the spectral efficiencies of the interferers transmitting on a common spectral resource, so that appropriate interference regimes could be implemented reliably at each receiver side, while ensuring that the total spectral efficiency of the system obtained after interference processing was maximized. The conducted optimization first led to a low-complexity interference regime algorithm, with only two regimes for each interferer, that allowed to identify the optimal interference regimes and spectral efficiencies to be used in any possible SNR/INR configuration of the 2-GIC.

Based on this result, we proposed to extend the present analysis to a scenario with $M = 2$ coalitions of $N > 1$ users assigned to each AP. A second optimization problem was considered: it consisted of a bijective one-to-one matching problem, whose objective was to form pairs of interferers. We assumed that any pair of interferers would then adapt their spectral efficiencies and interference regimes according to the previous IC. The optimal matching of interferers was then the one maximizing the total spectral efficiency of the system, after interference processing. We proposed an optimal algorithm allowing to compute the optimal matching. Finally, numerical Monte-Carlo simulations provided interesting insights on how coupling both IC and interferers matching could theoretically enhance the total spectral efficiency by a notable amount compared to state-of-the-art RRM techniques.

The analysis of the matching problem in a scenario with $M > 2$ coalitions of users was also investigated. However, it presented two limits: i) proposing an IC in a M-users Gaussian interference problem is too complex and still remains an open question at the moment; and ii) the matching problem, became a M-dimensional Assignment Problem, that is known to be NP-Hard when $M > 2$. Nevertheless, the matching problem was considered in a scenario with no IC: the interference was only processed as an additive source of noise and two suboptimal algorithms were proposed for approaching the optimal matching. The performance, in terms of total spectral efficiency of the presented algorithms also revealed notable performance gains compared to state-of-the-art RRM techniques.

The presented works could be enhanced by taking into the following enhancements that we leave for future work:

- **Interference Classification in M-GIC:** When the number of interferers coupled together was $M = 2$, we could define the best way to process the interference at each receiver side, so that the total spectral efficiency for the couple of interferers was maximized. However, when the number of interferers coupled together became larger than 2 (i.e. $M > 2$), defining the best interference regime becomes a lot more complex. Several papers, as [88], have tried to tackle the problem of both defining the best way to process interference in a multiple interferers scenario and estimating the system inherent spectral efficiency. Group SIC, iterative k-SIC or k-Joint Decoding approaches might also be considered as in [90], leading to multiple new regimes. Investigating these regimes requires to take into account the fact that decoding interference signals at each receiver is affected by the joint effect of interference, rather than each interfering signal. We conclude that it is then better to consider directly the effect of the combined interference signal. Recently, interference alignment techniques have been proposed and are based on this principle. The use of these techniques leads to achievable spectral efficiencies that, in some cases, can be as good as those over the 2-GIC [91, 92, 93]. However, these techniques remain theoretical and does not suit well practical implementation, which is the reason why we have not considered them in the conducted optimization. An extension of the presented work, assuming an IC for the M-users ($M > 2$) Gaussian interference channel could be of interest and could highlight new potential gains. The proposed algorithms used for matching could remain the same, as long as it is possible to define the performance tensor C, according to IC. We leave this open question for now, but will investigate in the next chapter a suboptimal approach to IC in a M-GIC.

- **NP-Hardness of the MAP and suboptimal matching:** When $M > 2$, the matching problem becomes NP-Hard. The suboptimal algorithms we considered have no guarantee of being the best performing algorithms (in terms of optimality, computation speed,etc.). Further investigation might lead to a more efficient matching algorithm, for solving the M-dimensional Assignment Problem, when $M > 2$.

- **Extension of the conducted analysis to different utility functions:** The conducted analysis has the objective of maximizing the total performance of the system, i.e. the total spectral efficiency after interference processing. Several other utility functions could have been considered, as in [86], such as: maximizing the minimal spectral efficiency offered to each user after interference processing, maximizing a weighted spectral efficiencies sum, maximizing the total spectral efficiency of a specific set of users, etc. Modifying the utility function would require to conduct the IC analysis and matching problems again, probably leading to different results.

- **Performance gains and network realization dependency:** In this chapter, we have not harnessed the expression of either the IC or the matching performance gains. In particular, it is easy to observe that when the channels are constant and equal for all the interferers in the system, any bijective matching appears to be optimal: the matching gain is then null in this extreme scenario. The matching performance gains depends on the heterogeneity of interferers in the system. In particular, it is easy to observe that when no heterogeneity exists (i.e. all interferers have the same SNRs/INRs), all matchings return the same performance, thus negating the potential performance offered by finding the optimal matching of interferers compared to a random matching procedure Similarly, the IC potential gain relies only on the possibility for some pairs of interferers to implement SIC-based interference mitigation techniques. Understanding how both potential gain evolve with the channel realizations is a complex task that we do not investigate throughout this chapter.

- **AP-UE Assignments were pre-defined:** In this chapter, we have assumed that the interferers were already pre-assigned to an AP. Allowing the system to re-assign its user to the APs might eventually lead to new potential gains. As a matter of fact, we investigate this possibility and the new potential gains that come along, in the next chapter, Chapter 6.

Chapter 6

Virtual Handover, Interference Classification and Interference Matching

6.1 Introduction

In Chapter 5, we have investigated a system with M Access Points (APs) and coalitions of users assigned to each AP. We investigated a Radio Resource Management (RRM) technique which methodically formed groups of 'friendly' interferers, so that interference classification (IC) could be exploited in order to enhance the global system performance. In the previous chapter that the User Equipments (UEs) were already pre-assigned to the APs. Within this chapter, we first introduce the concept of 'Virtual Handover' (VH): the AP providing the best SNR is not necessarily the best AP, in terms of spectral efficiency obtained after interference processing, when IC is considered. For this reason, it makes sense to let the system decide by itself how the UEs must be assigned to the APs. Based on this, we propose to extend the previous optimization problem, by considering that the optimal AP-UE assignments must be defined as well as the optimal interference regimes, spectral efficiencies and interferers matching. First, an updated version of the '2-Regimes Interference Classification' proposed in the previous chapter is derived: it leads to an algorithm which allows to define the optimal AP-UE assignments, interference regimes and spectral efficiencies to be used in a 2-users GIC. This algorithm is then exploited in the matching problem, in the case of $M = 2$ APs. Then, the extension to the $M > 2$ APs and coalitions is considered. In the previous chapter, we mentioned two issues, regarding both IC in M-users GIC and NP-Hardness of the matching problem with $M > 2$ APs. To address these issues, we first suggest a suboptimal game-theoretic IC in the M-users Gaussian Interference Channel (M-GIC), which allows to define interference regimes and spectral efficiencies to be used in any M-users GIC, with fixed AP-UE assignments. Then, we propose a genetic algorithm capable of solving the twofold matching problem, which consists of defining both the AP-UE assignments as well as the interferers matching. Finally, we conclude the chapter with numerical simulations that provide insights on the performance gains offered by coupling IC, interferers matching and AP-UE matching, in terms of total spectral efficiency.

The remainder of this chapter is organized as follows. After introducing the motivations, contributions and related works, we explain in Section 6.2, using an illustrative example, the reason why the AP-UE assignments must be redefined when the IC is considered. A novel interference classifier for the 2-users Gaussian Interference Channel (2-GIC) is then derived, from the previous '2-regimes interference classification', which is capable of defining the optimal interference regimes, spectral efficiencies and AP-UE assignments for any 2-users GIC, with unassigned UEs. Section 6.3 introduces the extension of the matching problem, with $M = 2$ APs and NM unassigned users. The conducted analysis reveals that the problem can be optimally solved, by considered a graph theory approach, using a weighted form of the Edmonds algorithm. In Section 6.4, we study the extension of the problem, to the scenario with $M > 2$ APs. More specifically, we address the problem of IC in a M-GIC, with assigned UEs, in Section 6.4.2, by proposing a game-theoretic suboptimal approach. In Section 6.4.3, we investigate the twofold matching (defining both the matching of interferers and the matching of UEs to APs). It turns out that the problem falls into a class of Non-Linear Programming problems, which are known to be NP-Hard. To address this issue, we propose a suboptimal genetic algorithm, which is extensively detailed. At the end of both Sections 6.3 and 6.4, numerical simulations provide insights on the threefold performance gains offered by IC, interferers matching and AP-UE re-assignments. Finally, Section 6.5 concludes the chapter.

6.1.1 Related Works and Contributions

In the previous chapter, we proposed a RRM technique, exploiting the recent advances in IC. In a 2-GIC, we proposed to adapt the spectral efficiencies and the interference regimes of each interferer so that the total spectral efficiency obtained after interference processing, at each receiver side, was maximized. Furthermore, we extended the problem, by considering a matching problem where $M \geq 2$ coalitions of $N \geq 1$ interferers had to be matched together. The interferers in a group transmitted on the same spectral resource, thus interfering, but we assumed that they would then implement interference regimes and spectral efficiencies, according to our previous IC. The objective was then to find the optimal one-to-one matching of interferers, so that the total spectral efficiency of the system was maximized. The conducted analysis revealed interesting insights about the potential performance gains that could be obtained via IC and interferers matching. However, the presented analysis exhibited two limitations, that we address in this chapter.

First, when more than two coalitions had to be matched together, a M-GIC had to be considered and defining the best interference regimes became a lot more complex. Group SIC, iterative k-SIC or k-Joint Decoding approaches might also be considered as in [90], leading to multiple new regimes. Investigating these regimes requires to take into account the fact that decoding interference signals at each receiver is affected by the joint effect of interference, rather than each interfering signal. Therefore, it is better to consider directly the effect of the combined interference signal. Recently, interference alignment techniques have been proposed that are based on this principle. The use of these techniques leads to achievable spectral efficiencies that, in some cases, can be as good as those over the 2-GIC [91, 92, 93]. However, these techniques remain theoretical and does not suit well practical implementation, which is the reason why we have not considered them in the conducted optimization. In this chapter, we address the problem of defining interference regimes and spectral efficiencies after interference processing in any M-GIC, by considering a non-cooperative game, where each interferer can adapt its own spectral efficiency and interference regime, with the objective of maximizing its own spectral efficiency after interference processing.

Second, we break the previous assumption that the UEs were already pre-assigned to an AP. We show, in this chapter, that allowing the system to re-assign the UEs to the APs can lead to new potential gains. As a consequence of these results, it appears that when IC is considered, the AP providing the best SNR is not always the best option. The optimization process must then take into account the possibility of re-assigning the NM users to the M APs. More specifically, under certain configurations, the system benefits from reassigning a user from the AP providing the best SNR to another one with a lower SNR, as it might lead to a better spectral efficiency after interference processing. Also, allowing the system to re-define the AP-UE assignments provides an additional degree of multiuser diversity. We extend the previous optimization, by assuming that the users deployed in the system are now unassigned and that their AP-UE assignments have to be defined in the optimization analysis. As suggested in Figure 6.1, the optimization is now threefold:

1. Define the assignments of the NM UEs to M AP and form of M coalitions of N UEs assigned to each AP.

2. Define the matching of interferers and form N groups of M interferers: each group of interferers forms a M-users GIC.

3. Finally, define the spectral efficiencies and interference regimes, in each M-users GIC.

As before, the objective of the threefold optimization consists of defining the optimal configuration, that maximizes the network performance, i.e. the total spectral efficiency.

Our contributions in this chapter can be summed up as follows:

- First, we demonstrate that RRM problems, exploiting IC, must take into account the AP-UE assignments: the classical paradigm, which assigns each user to the AP providing the best SNR made sense when interference was treated as noise exclusively, but does not hold, when IC is considered. We named this concept 'Virtual Handover', which is extensively detailed in Section 6.2.2

- Assuming the system might exchange the AP-UE assignments, we derive a new IC algorithm for 2-users GIC, from the previous 2-regimes IC that we proposed in Section 5.3. It leads to a new classifier that is able to define for any 2-GIC with unassigned users, the optimal AP-UE assignments, spectral efficiencies and interference regimes, maximizing the total spectral efficiency.

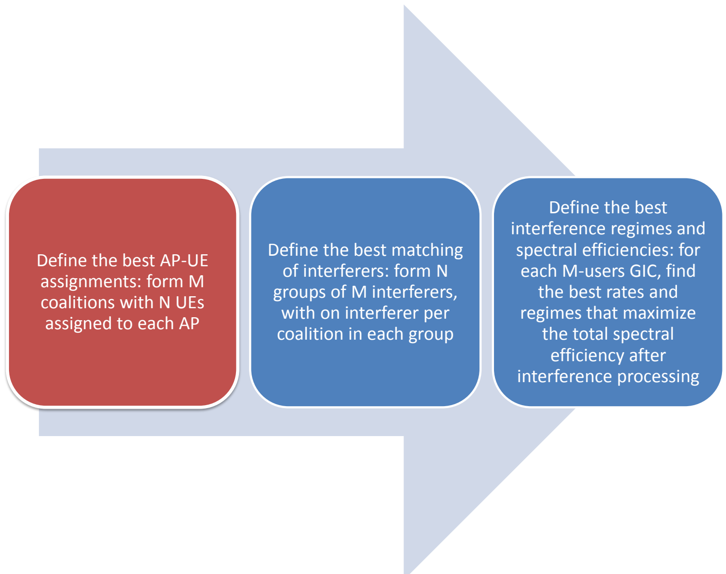

Figure 6.1: The threefold optimization problem. The AP-UE assignment comes as an additional layer to the previous optimization problem, which only included matching of interferers and IC.

- This updated classification is then exploited in an updated version of the matching problem detailed in Section 5.4, now allowing the system to re-assign the users to the available $M = 2$ APs, at will. In Section 6.3, we conduct the analysis of the optimization problem and reveal that it is strictly equivalent to a problem of a maximum weight disjoint edge matching in a $2N$ complete graph. For such problems, optimal algorithms exist, namely the Edmonds algorithm [94, 102, 103]. Numerical simulations reveals that an additional gain is offered to the system when it is given the possibility of reassigning its users to the APs.

- We also propose a novel game-theoretic approach for IC in M-users GIC. Even though suboptimal, the IC we proposed leads to notable results and can be reused in the extension of the matching problem with $M > 2$ APs and coalitions of users, previously detailed in Section 5.5. It is discussed in Section 6.4.2.

- Finally, we investigate, in Section 6.4, the threefold optimization problem, when $M > 2$, observe that it is equivalent to an Non-Linear Programming (NLP) problem, which is known to be NP-Hard [107]. We propose and detail extensively a specifically adapted suboptimal genetic algorithm, that is able to compute a suboptimal solution to the considered optimization problem.

The content of this chapter has been published in three papers. The first one introduces the concept of 'Virtual Handover' and investigates the extension of the IC and matching problem with AP-UE re-assignments, in the $M = 2$ APs scenario [118]. The second paper details the game-theoretic approach for IC in M-GIC, which is detailed in Section 6.4.2 [120]. The presented concepts about 'Virtual Handover' have also been filed in a patent [121].

6.2 Extension of the Previous Interference Classification

6.2.1 System Model and Optimization Problem: Reminder of the Previous Results

In this section ans as in Section 5.3, we consider a 2-GIC, consisting of $M = 2$ APs and $N = 2$ UEs (i.e. 1 UE per AP). For any $i \in \{1, 2\}, k \in \{1, 2\}$, we denote $h(k, i)$ the channel between AP k and UE i. If the AP k is used for transmission to UE i, the combination AP k and UE i can be referred to as 'interferer (k, i)'. We denote $p(k)$ the transmission power used by AP k to transmit to UE i. For now, we also assume that the transmission powers are fixed and that $\forall k \in \{1, 2\}$, $\forall i \in \{1, 2\}$, $p(k, i) > 0$. Finally, the noise is assumed to be Gaussian, with variance σ_n^2. We also assume that each UE is assigned to one and only one of the M AP. This assumption allows for a simple mathematical problem with low complexity, and we showed in the previous chapter that it helps the matching process detailed hereafter converge to a solution. In the previous chapter, we also assumed that the AP-UE assignments were pre-established, and $\forall i \in \{1, 2\}$, UE i was assigned to AP i. We also denote ω the set of all signal to noise ratios $\gamma(i, j)$ related to the two interferers:

$$\forall (i, j) \in \{1, 2\}^2,\ \gamma(i, j) = \frac{p(j, i)|h(j, i)|^2}{\sigma_n^2} \tag{6.1}$$

$$\omega = (\gamma(1, 1), \gamma(2, 2), \gamma(1, 2), \gamma(2, 1)) \tag{6.2}$$

As before, we assume that the APs cooperate, that perfect knowledge of the transmission settings are given to the APs and that each interferer (i, i) is able to process interference according to 3 possible regimes. We denote O_i the interference regime of interferer (i, i) which can take 3 values, corresponding to the 3 possible regimes:

Noisy - $O_i = 1$: the interference is weak enough to be processed as additive noise at the receiver side.

Successive Interference Cancellation (SIC) - $O_i = 3$: UE i can decode the strong incoming interference and cancel it out of the received signal using SIC.

Orthogonalization - $O_i = 2$: interferer (i, i) attempts to avoid interference by transmitting using only the i-th half of spectral resources. If interferer (j, j), with $j \neq i$, performs orthogonalization as well, interference is avoided, at the cost of using only half of the spectral resources.

And each regimes combination $\mathcal{O} = (O_1, O_2)$ leads to maximal spectral efficiencies for both users, as summed up in Table 5.2, whose sum corresponds to the total spectral efficiency for the pair of users. When attempting to maximize the total spectral efficiency, we detailed in Section 5.3, that the analysis of the optimization problem leads to a classifier with only two possible regimes O_i per interferer, as summed up in Proposition 5.7, leading to only 4 possible regimes combinations $\mathcal{O}$, namely $(1, 1)$, $(1, 3)$, $(3, 1)$ and $(3, 3)$.

6.2.2 Illustrative Example of Virtual Handover

In this section, we provide an illustrative example, showing that the system is not always interested in the pre-defined AP-UE assignments and might instead want to re-assign its UEs to different APs. More specifically, lets us consider the scenario depicted in Figure 6.2. We focus on a single user, being interfered by an interference with constant INR value denoted INR (due to an external concurrent transmission happening at a spectral efficiency R_I, and consider two APs with two respective SNRs, denoted SNR_1 and SNR_2, we might wonder whose AP the user should be assigned to, in order to maximize the spectral efficiency obtained after interference processing. When interference is treated as noise exclusively, which is the case in every single RRM problem not considering IC, the answer is simple: the most suitable AP is the one that provides the best SNR. Indeed, when the interference is treated as noise, the spectral efficiency obtained after interference processing only depends on the SINR. Since the INR is constant, the best AP is then the one which provides the best SNR. However, if we now consider IC, and the possibility of treating interference according to a SIC-based regime, this result does not necessarily hold anymore. Without loss of generality, let us now consider that $SNR_1 > SNR_2$. If interference is treated as noise, the best AP is AP 1. Since AP 2 has a lower SNR, it might allow the user to decode the incoming interference, and it could probably not have done it if it was assigned to AP 1, since it treated interference as noise. This happens if and only if:

$$\log_2\left(1+\frac{INR}{1+SNR_2}\right) \geq R_I > \log_2\left(1+\frac{INR}{1+SNR_1}\right) \tag{6.3}$$

In such a configuration, the user is able to decode and cancel the interference if it is assigned to AP 2, but not if it is assigned to AP 1. The spectral efficiency obtained after interference processing when assigned to AP 1 is then $R_1 = \log_2\left(1+\frac{SNR_1}{1+INR}\right)$. The one obtained after interference processing, when assigned to AP 2, is then $R_2 = \log_2(1+SNR_2)$. In such a configuration, we can have $R_2 > R_1$, even if $SNR_1 > SNR_2$, which happens if:

$$SNR_1 > SNR_2 > \frac{SNR_1}{1+INR} \tag{6.4}$$

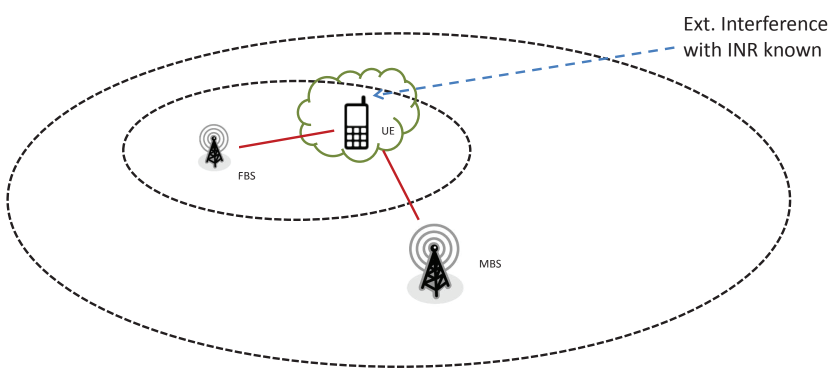

Figure 6.2: Illustrative example of 'Virtual Handover': assigning the UE to another AP, can provide a better spectral efficiency after interference processing, even with a smaller SNR.

This example we called 'Virtual Handover' demonstrates that considering IC forces the system to reconsider its classical AP-UE assignments procedure:

assigning the UEs to the APs providing the best SNR is not always the best option. There are in fact numerous scenarios where the system might benefit from assigning a UE to another AP, providing a non-maximal SNR, as suggested in [100]. For this reason, we propose, in this section, to re-investigate the conducted analysis detailed in Chapter 5, assuming that the users are no longer pre-assigned and leave it to the system to define the best AP-UE assignment, as well as the best spectral efficiencies, interference regimes and interferers matching, leading to a threefold optimization problem we summed up in Figure 6.1.

6.2.3 Including AP-UE Assignments: Update on the Interference Regimes

We first focus on the case of a network with $M = 2$ APs and $NM = 2$ unassigned UE. Assuming we know the SNR/INR configuration ω of the 2-GIC, our objective in this section is to define the best assignment, interference regimes and spectral efficiencies for each user, so that the total spectral efficiency is maximized. The two interferers can be assigned to any of the AP, but share the same spectral resources: they may suffer from interference, but can treat it according to the 3-regimes IC we defined previously in Section 5.2. Let us first define the updated notation $\mathcal{O}$, which now allows to define both the possible interference regimes and AP-UE assignments combinations.

- $\mathcal{O} = (O_1, O_2)$ refers to the configuration where AP 1 (resp. AP 2) is assigned to UE 1 (resp. UE 2) and the interference regime for UE 1 (resp. UE 2) is O_1 (resp. O_2).
- $\mathcal{O} = (O_1, O_2)^*$ refers to the configuration where AP 1 (resp. AP 2) is assigned to UE 2 (resp. UE 1) and the interference regime for UE 1 (resp. UE 2) is O_1 (resp. O_2).
- $\mathcal{O} = (2, 2)^i$ refers to the configuration where both UEs are assigned to AP i: in this configuration, we assume they equally split the available spectral resources and transmit avoiding interference (then leading to both interference regimes being $O_1 = O_2 = 2$).

Based on this notation, we can define 10 possible configurations for $\mathcal{O}$, and maximal spectral efficiencies $R_i(\mathcal{O}, \omega)$ for each UE $i \in \{1, 2\}$ and for any SNR/INR configuration ω. We listed them in Table 6.1. Note that we removed the $(2,3)$, $(3,2)$, $(2,3)^*$, $(3,2)^*$, $(2,2)$ and $(2,2)^*$ configurations, as we were able to prove in Section 5.2 that they were always outperformed.

$\mathcal{O}$	$R_1(\mathcal{O},\omega)$	$R_2(\mathcal{O},\omega)$
$(1,1)$	$\log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right)$	$\log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right)$
$(3,1)$	$\log_2\left(1+\gamma(1,1)\right)$	$\log_2\left(1+\min\left[\frac{\gamma(2,2)}{1+\gamma(1,2)},\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$
$(1,3)$	$\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)},\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$	$\log_2\left(1+\gamma(2,2)\right)$
$(3,3)$	$\log_2\left(1+\min\left[\gamma(1,1),\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$	$\log_2\left(1+\min\left[\gamma(2,2),\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$
$(1,1)^*$	$\log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right)$	$\log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right)$
$(3,1)^*$	$\log_2\left(1+\gamma(2,1)\right)$	$\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)},\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$
$(1,3)^*$	$\log_2\left(1+\min\left[\frac{\gamma(2,2)}{1+\gamma(1,2)},\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$	$\log_2\left(1+\gamma(1,2)\right)$
$(3,3)^*$	$\log_2\left(1+\min\left[\gamma(2,1),\frac{\gamma(2,2)}{1+\gamma(1,2)}\right]\right)$	$\log_2\left(1+\min\left[\gamma(1,2),\frac{\gamma(1,1)}{1+\gamma(2,1)}\right]\right)$
$(2,2)^1$	$\frac{1}{2}\log_2\left(1+\gamma(1,1)\right)$	$\frac{1}{2}\log_2\left(1+\gamma(1,2)\right)$
$(2,2)^2$	$\frac{1}{2}\log_2\left(1+\gamma(2,1)\right)$	$\frac{1}{2}\log_2\left(1+\gamma(2,2)\right)$

Table 6.1: The 10 admissible configurations $\mathcal{O}$ and their spectral efficiencies performances $R_i(\mathcal{O},\omega)$ for each UE $i \in \{1,2\}$.

In order to pursue the analysis , let us recall the $\triangleright$ operator, where $\mathcal{O}\triangleright\mathcal{O}'$ means that the configuration $\mathcal{O}$ offers a better maximal total spectral efficiency $R(\mathcal{O},\omega)$ than $\mathcal{O}'$, when the SNR/INR configuration is ω:

$$\mathcal{O}\triangleright\mathcal{O}' \Leftrightarrow R(\mathcal{O},\omega) \geq R(\mathcal{O}',\omega) \tag{6.5}$$

Our objective in this section is twofold: first, we identify configurations of interest, i.e. configurations $\mathcal{O}$ that can potentially be the best performing configurations for certain realizations of ω; and second, we define criteria on ω that immediately tell which configuration of interest $\mathcal{O}$ is the best performing configuration. Let us now consider the two following propositions 6.1 and 6.2 that allow for simplifications.

Proposition 6.1. *For any given channel realization ω and any configuration* *inducing orthogonalization on both sides, there exists a configuration that outperforms it. More precisely:*

- $(2,2)^1$ *is outperformed by either* $(2,3)^*$, $(3,2)$.

- $(2,2)^2$ *is outperformed by either* $(3,2)^*$, $(2,3)$.

Proof. Refer to Appendix 8.10 for proof. □

Proposition 6.2. *In scenarios, where both users can decode and cancel interference using SIC-based techniques, it is more interesting for the system to transmit using the interfering links, instead of its pre-assigned ones and treat interference as noise, i.e.,* $\forall\omega$*:*

- $(1,1)^* \rhd (3,3)$.

- $(1,1) \rhd (3,3)^*$.

Proof. Refer to Appendix 8.11 for proof. □

Based on the previous propositions, we demonstrate that our classifier only operates within 6 configurations of interest, namely $(1,1)$, $(1,3)$, $(3,1)$, $(1,1)^*$, $(1,3)^*$ and $(3,1)^*$. Also, no orthogonalization-based configuration subsists, as they are all outperformed by at least one of the 6 configurations of interest. As in the previous '2-Regimes Interference Classifier' from Section 5.2, each UE i can only treat interference according to 2 interference regimes: $O_i = 1$ (Noisy) or $O_i = 3$ (SIC). Moreover, the AP-UE assignments of each configuration guarantee that each AP is assigned one and only one UE. We now focus on defining, for any SNR/INR configuration ω, the Best Performance Configuration (BPC) $\mathcal{O}$, that outperforms all the other configurations. The '6 Configurations Classifier' defined in Proposition 6.3 returns, for any channel configuration ω, the AP-UE assignment and the interference regimes configuration $\mathcal{O}$, corresponding to the best performing configuration.

Proposition 6.3. *We define the '6 Configurations Classifier', as follows:*

1. *If* $\gamma(1,1) \geq \gamma(1,2)$ *and* $\gamma(2,2) \geq \gamma(2,1)$

   - $(1,1)$ *BPC* $\Leftrightarrow \gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$ *and* $\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$
   - $(1,3)$ *BPC* $\Leftrightarrow (1,1)$ *not BPC and* $\gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)$
   - $(3,1)$ *BPC* $\Leftrightarrow (1,1)$ *not BPC and* $\gamma(2,2)+\gamma(1,2) \leq \gamma(1,1)+\gamma(2,1)$

2. If $\gamma(1,1) \leq \gamma(1,2)$ *and* $\gamma(2,2) \leq \gamma(2,1)$

- $(1,1)^*$ *BPC* $\Leftrightarrow \gamma(2,1) \geq \gamma(2,2)(1+\gamma(1,1))$ *and* $\gamma(1,2) \geq \gamma(1,1)(1+\gamma(2,2))$

- $(1,3)^*$ *BPC* $\Leftrightarrow (1,1)^*$ *not BPC and* $\gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)$

- $(3,1)^*$ *BPC* $\Leftrightarrow (1,1)^*$ *not BPC and* $\gamma(2,2)+\gamma(1,2) \leq \gamma(1,1)+\gamma(2,1)$

3. If $\gamma(1,1) \geq \gamma(1,2)$ *and* $\gamma(2,2) \leq \gamma(2,1)$

- $(3,1)$ *BPC* $\Leftrightarrow \frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ *and*

$$(1+\gamma(1,1))(1+\gamma(2,2) \geq (1+\gamma(2,1))(1+\gamma(1,2))$$

- $(3,1)^*$ *BPC* $\Leftrightarrow \frac{\gamma(2,1)}{1+\gamma(1,1)} \geq \frac{\gamma(2,2)}{1+\gamma(1,2)}$ *and*

$$(1+\gamma(1,1))(1+\gamma(2,2) \leq (1+\gamma(2,1))(1+\gamma(1,2))$$

4. If $\gamma(1,1) \leq \gamma(1,2)$ *and* $\gamma(2,2) \geq \gamma(2,1)$

- $(1,3)^*$ *BPC* $\Leftrightarrow \frac{\gamma(1,1)}{1+\gamma(2,1)} \leq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ *and*

$$(1+\gamma(1,1))(1+\gamma(2,2) \leq (1+\gamma(2,1))(1+\gamma(1,2))$$

- $(1,3)$ *BPC* $\Leftrightarrow \frac{\gamma(2,1)}{1+\gamma(1,1)} \leq \frac{\gamma(2,2)}{1+\gamma(1,2)}$ *and*

$$(1+\gamma(1,1))(1+\gamma(2,2) \geq (1+\gamma(2,1))(1+\gamma(1,2))$$

Proof. The detailed proof of this proposition is given in Appendix 8.12. □

We have then defined an updated version of our previous '2-Regime Interference Classifier' from Section 5.2, that is able to return, for any configuration of the 2-GIC, the configuration $\mathcal{O}$, i.e. the interference regimes, the spectral efficiencies and the AP-UE assignments for each UE, that maximizes the total spectral efficiency after interference processing.

6.3 Interference Classification, Matching and Assignments: The $M = 2$ Scenario

6.3.1 System Model and Optimization Problem

In this section, we consider the matching problem with $M = 2$ APs and $2N$ unassigned UEs (we $N > 1$ UE per AP), which is the extension of the matching problem presented in Section 5.4, when the system is able to select the most appropriate AP-UE assignments. Based on the previous analysis, the matching problem would consist of dividing the $2N$ users in N groups of 2 UEs. Each UE in each group will be assigned to a different AP. Assuming we have split the available resources in N equal Spectral Resources Elements (SREs) $(G_i)_{i\in\mathcal{N}}$, every group $i$is assigned to spectral resource G_i: no interference exists between users from different groups, as they are transmitting on different spectral resources. The interferers belonging to a same group, are assigned to different APs, are sharing a same spectral resource, and suffer from interference. However, they can implement interference regimes, adapt their spectral efficiency and define the best AP-UE assignments according to our previous '6 Configurations Classifier', defined in Proposition 6.3 and maximize the total spectral efficiency for the group of interferers. By doing so on each group, the system is able to maximize the total spectral efficiency of the system. Our conducted analysis leads to a twofold optimization problem, where the objective is to maximize the total spectral efficiency of the system i) by forming N groups of 2 interferers and ii) by defining AP-UE assignments, as well as interference regimes, i.e. the best configurations $\mathcal{O}$ for every interferer in every group of interferers. The problem is addressed in a two-steps optimization. First, we observe that for any pair of interferers (i, j), our previous ' 6 Configurations Classifier' gives the best configuration to be used and the total spectral efficiency obtained after interference processing $R'(i, j, \omega(i, j))$ for the interferers in this group. $\omega(i, j)$ refers to the SNR/INR configurations related to the interferers i and j. The second step consists of finding the N disjoint pairs of interferers, i.e. the optimal interferers matching m^*, that maximize the total spectral efficiency of the system $\mathcal{R}$. This leads to the following optimization problem (6.6).

$$m^* = \arg\max_{m} \left[\mathcal{R} = \sum_{i=1}^{NM} R'(i, m(i), \omega(i, m(i)) \right] \tag{6.6}$$

Where

- $\forall i_1, i_2 \in \mathcal{N}' = \{1, ..., NM\}, i_1 \neq i_2, m(i_1) = i_2$ means that interferer i_1 is coupled with interferer i_2. If $m(i_1) = i_2$, then necessarily, $m(i_2) = i_1$.

- $\omega(i_1, i_2)$ plays the same role as ω in Section 6.2.3 and contains the SNR/INR γ elements related to UEs i_1 and i_2.

The maximal spectral efficiency $R'(i_1, m(i_1), \omega(i_1, m(i_1))$ that any UE pair (i_1, i_2) can access is given by our previous '6 Configurations Classifier': the classifier returns the optimal AP-UE assignment, the interference regimes for our pair of interferers, and the maximal spectral performances for our couple of interferers. We can then define a $2N \times 2N$ matrix C, whose general term is defined as:

$$C(i,j) = \begin{cases} -\infty & \text{if } i = j. \\ R'(i,j,\omega(i,j)) & \text{otherwise.} \end{cases} \tag{6.7}$$

The purpose of the $-\infty$ term is to discourage the system of matching an interferer i with itself. By doing so, we force the system to consider disjoint matchings, i.e. $\forall i, m(i) = j \neq i$.

We can then observe that our optimization problem (6.6) is actually strictly equivalent to finding the N disjoint pairs of interferers (or the assignments m^*) that maximize the total spectral efficiency $\mathcal{R}$. From the graph theory point of view, we can represent a $2N$ complete graph, where each node represents a UE and each edge between two distinct nodes $i$and j has a weight $C(i, j)$. The optimization now consists of finding a maximum weight disjoint edges matching, i.e. select N edges, with no two edges sharing a same node, such that the sum of edges is maximized, as depicted in Figure 6.3. This is a well-known graph theory problem, which is easily and optimally solved by a combinatorial low-complexity algorithm, namely the weighted Edmonds algorithm [94, 102, 103].

6.3.2 Numerical Simulations: $M = 2$ Case

In this section, we highlight the performance gains of our optimization approach, by running Monte-Carlo simulations, with $N_{MC} = 1000$ independent realizations. We have considered two APs, within a distance of d_{AP}. The $2N$ unassigned UEs are uniformly distributed in the coverage area of each AP R_{AP}. In the following, we denote $d(i, j)$ the distance between AP i and UE j. The channels $h(i, j)$ include the antenna gain G, the path loss $L(d(i, j))$ and the shadowing ξ. All parameters are summarized in Table 6.2, and are based on [188, 189].

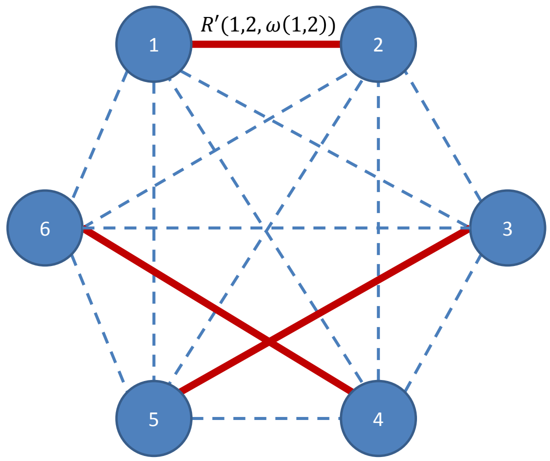

Figure 6.3: The maximum weight disjoint edges matching problem in a $2N$-complete graph. The red bold configuration is a possible disjoint matching, which matches interferers 1 and 2, 4 and 6, 3 and 5.

We consider 5 Interference Management Strategies (IMS) of interest:

- **IMS 1**: orthogonalization is performed so that no UEs are interfering and each UE is assigned to its best AP.
- **IMS 2**: AP-UE assignments are randomly defined, so that each AP has N assigned UEs, but prioritize the UEs closest to the APs. Interferers matching is also random. Interferers have no other choice but to treat incoming interference as an additive source of noise.
- **IMS 3**: AP-UE assignments are randomly defined, so that each AP has N assigned UEs, but prioritize the UEs closest to the APs. Interferers matching is also random. Interferers can treat interference according to the best admissible interference regime.
- **IMS 4**: AP-UE assignments are randomly defined, so that each AP has N assigned UEs, but prioritize the UEs closest to the APs. Best interferers matching is computed with Kuhn-Munkres algorithm previously detailed

Parameter	Value
Distance between AP to AP d_{AP}	750m
Coverage Area R_{AP}	Users are unif. dist. ,s.t. dist. AP-UE $\in [r_{min}, r_{max}]$
$[r_{min}, r_{max}]$	$[35m, 600m]$
Transmission powers $p_k(.)$	Proportional to dist. $\in [20dBm, 46dBm]$
Channels $h(i,j)$	$h(i,j) = \frac{G}{L(d(i,j))\xi}$
Antenna Gain G	10 dBi
Path Loss $L(d(i,j))$, [d in km]	$L = 131.1{+}42.8\log_{10}(d(i,j))$
Shadowing ξ	Log-normal, $\sigma_{SH} = 10$ dB
Noise power σ_n	-104 dBm
Number of unassigned UEs NM	50

Table 6.2: Simulations Parameters.

in Section 5.4.3. Interferers can treat interference according to the best admissible interference regime.

- **IMS 5**: Define the optimal configuration with optimal AP-UE assignments, interferers matching and interference regimes.

We present in Figure 6.4, the distribution of the spectral efficiency per user and the average performance in terms of spectral efficiency per user, for each IMS, over $N_{MC} = 1000$ independent Monte-Carlo simulations. We observe that full orthogonalization, i.e. *IMS 1*, is spectrally inefficient. This is an expected result that we have also observed in our previous matching problem, in the Section 5.4.4. We now consider as a reference *IMS 2*, where the assignments and matchings are both random, and interference is treated as additive noise. Optimizing the interference regimes, the interferers matchings and the AP-UE assignments lead to notable performance improvements. More precisely, allowing the system to select the best interference regime (*IMS 3*) offers an average performance improvement of 11.1%, compared to *IMS 2*. Furthermore, allowing the system to select the most appropriate interferers matching (*IMS 4*), leads to an average performance improvement of 16.8%. Finally, allowing the system to select the most appropriate UEs-APs assignments (*IMS 5*) allows an summed up average performance improvement of 28.0%, compared to *IMS 2*.

The overall gain between *IMS 2* and *IMS 5* can be decomposed in 3 parts:

- **Interference Classification Gain**: First, the gain offered by IC (between *IMS 2* and *IMS 3*). Even though hard to harness, this gain appears

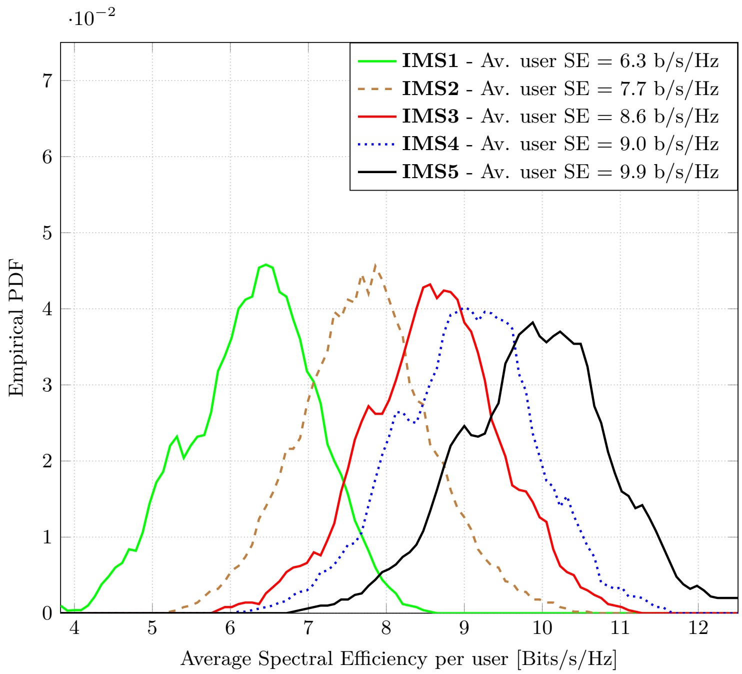

Figure 6.4: Histogram plot of the performances of each scenario, for $N_{MC} = 1000$ independent realizations ($M = 2$ interferers per group, $N = 25$ groups of interferers).

to depend on the proportion of interferers implementing a SIC regime, instead of a noisy one.

- **Interferers Matching Gain**: Secondly, there is a gain related to the matching of interferers (between *IMS 3* and *IMS 4*), which seems to depend on the variance of the SNRs $\gamma(i, j)$. As suggested in Section 5.6, it is easy to picture that there would be no gain between the best matching and the random matching if the INRs/SNRs $\gamma(i, j)$ were the same for every interferer. Harnessing this gain explicitly, based on the SNRs γ is however complicated.

- **AP-UE Assignments Gain**: Finally, there also appear to be a new notable gain, related to the capability offered to the system to assign the UEs to more appropriate APs, and not necessarily the one providing the best SNR (between *IMS 4* and *IMS 5*). This gain also seems to scale with the diversity of AP offered to any interferer of the system, but it needs further investigation, as it is quite complex to explicitly define it.

6.4 Interference Classification, Matching and Assignments: The $M > 2$ Scenario

6.4.1 System Model and Optimization Problem

Let us now consider the extension of the matching problem with $M > 2$ APs in the network. The matching procedure now splits the NM unassigned UEs into N groups of M interferers. In each group of interferer, every single user is assigned to a different AP. The APs transmit over the same spectral resource for users belonging to a same group, thus leading to interference between these transmissions. We have demonstrated in Section 6.2 that we could define the best configuration (i.e. interference regimes and AP-UE assignments) in any SNR/INR configuration ω of the 2-GIC, according to our '6 Configurations Classifier'. However, and for the same reasons we pointed out in Section 5.6, discussing about IC in every group, which consists of a single M-GIC rapidly becomes impossible when $M > 2$. In the following Section 6.4.2, we discuss a possible way to define interference regimes and spectral efficiencies for any M-GIC. However suboptimal, the approach we propose allows to take into account SIC-based interference mitigation techniques when they allow a user to enhance its spectral efficiency without constraining the spectral efficiency of the interferer whose interference is decoded.

Let us now consider the following two matching notations m and u, where:

$$\forall i \in \mathcal{M} = \{1, ..., M\}, \forall j \in \mathcal{N}' = \{1, ..., MN\}, m(i,j) = \begin{cases} 1 & \text{if UE } j \text{ is assigned to AP } i \\ 0 & \text{else} \end{cases} \tag{6.8}$$

$$\forall i \in \mathcal{N} = \{1, ..., N\}, \forall j \in \mathcal{N}', u(i,j) = \begin{cases} 1 & \text{if UE } j \text{ is assigned to group of interferers } i \\ 0 & \text{else} \end{cases} \tag{6.9}$$

The matchings m and u are also constrained to guarantee that each UE is assigned to exactly one AP and one group, and that there are no two UEs in a same group assigned to the same AP. This is strictly equivalent to the following set of constraints:

$$\forall j \in \mathcal{N}', \sum_{i=1}^{M} m(i,j) = 1 \tag{6.10}$$

$$\forall j \in \mathcal{N}', \sum_{i=1}^{N} u(i,j) = 1 \tag{6.11}$$

$$\forall i \in \mathcal{M}, \forall k \in \mathcal{N}, \forall j, j' \in \mathcal{N}', j \neq j', u(k,j)+u(k,j')+m(i,j)+m(i,j') \leq 3 \tag{6.12}$$

Given a set of APs, UEs and their SNRs $\Gamma = (\gamma(i,j))_{i\in\mathcal{M},j\in\mathcal{N}'}$, the matching problem consists then of finding the optimal matchings u^* and m^*, among all the possible matchings satisfying the previous constraints, that maximize the total spectral efficiency of the system $\mathcal{R}(m,u,\Gamma)$, which is defined as:

$$\mathcal{R}(m,u,\Gamma) = \sum_{i=1}^{M}\sum_{j=1}^{MN}\sum_{k=1}^{N} m(i,j)u(k,j)R(j,m,u,\Gamma) \tag{6.13}$$

Where $R(j,m,u,\Gamma)$ denotes the spectral efficiency that user j can benefit from, when UE j is assigned to the AP given by m and is grouped with interferers, according to u. Computing the spectral efficiency for this UE is detailed more extensively in the following section.

6.4.2 A Game-Theoretical Approach to Interference Regimes in the M-users Gaussian Interference Channel

A given instance of matchings m and u, allows to form N groups of M UEs and defines the AP-UE assignments for these UEs. Every single UE $j \in \{1,...,NM\}$ is involved in a M-GIC, with $N-1$ other UEs and their respective SNRs/INRs can be defined thanks to m, u and Γ. Our objective here is to identify the maximal spectral efficiency $R(j,m,u,\Gamma)$ that each interferer in this M-GIC can pretend to, without outage, while implementing interference regimes from our previous IC. Defining the optimal interference regimes in a M-GIC is complex, we propose in thus section a suboptimal IC based on the only two regimes that emerged in our previous IC: the noisy regime and the SIC-based regime. Group SIC, iterative k-SIC approaches are then considered, as in [90], leading to multiple new regimes.

For simplicity, let us adapt the notations in this section only. Let us assume simply consider a M-GIC, with M UEs $i \in \mathcal{M} = \{1, ..., M\}$ being respectively assigned to APs i. We also denote $\gamma(i, j)$ the signal to noise ratio concerning the signal emitted by AP i and perceived by UE j, which can be obtained for any matching sets (m, u) from Γ. The M-GIC that we consider in this section, is then simply represented in Figure 6.5.

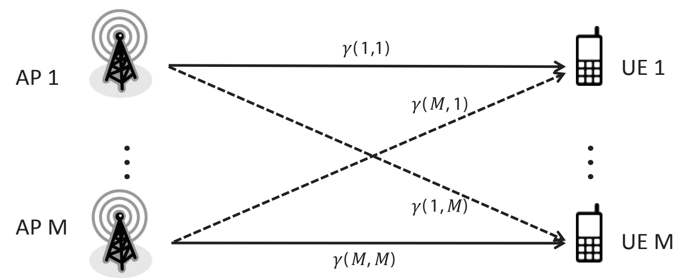

Figure 6.5: The general M-GIC considered in this section.

We consider that each AP-UE pair i can adapt its spectral efficiency R_i at will and may process the interference coming from the $M-1$ other pairs, by attempting to decode it and cancel it via SIC if possible, otherwise process it as noise. The objective for each AP-UE pair i is to maximize the spectral efficiency obtained after interference processing S_i, by adapting its own spectral efficiency R_i, at which it attempts to transmit.

$$R_i^* = \arg\max_{R_i} \left[S_i(R_i, R_{-i}) \right] \tag{6.14}$$

Where R_{-i} denotes the spectral efficiencies selected by the other $M-1$ interferers. In that sense, we have defined a simple M-users non-cooperative game. The spectral efficiency obtained after interference processing $S_i(R)$ must however be defined. According to our previous assumptions, we can define the maximal spectral efficiency after interference processing for user i, denoted $\epsilon_i(R_{-i})$, as the spectral efficiency obtained when all the possible interferers, that could have been decoded, have been canceled out of the received signal of user i, whereas the undecodable interference is treated as noise. In such a context, if the spectral efficiency R_i is higher than the maximal spectral efficiency after interference processing $\epsilon_i(R_{-i})$ that this user could pretend to, an outage happens: the user is unable to decode its transmission and the spectral efficiency obtained after interference processing S_i is then nullified.

$$S_i(R_i, R_{-i}) = \begin{cases} 0 & \text{if } R_i > \epsilon_i(R_{-i}) \\ R_i & \text{else} \end{cases} \tag{6.15}$$

In the game we designed, there is a balance between the different spectral efficiencies of the interferers R_i:

- Every user can adapt its spectral efficiency, and wishes to maximize the spectral efficiency it obtains after interference processing S_i, which increases with R_i, until it reaches $\epsilon_i(R_{-i})$, as depicted on figure 6.6
- The maximum threshold $\epsilon_i(R_{-i})$ is a convex function of $R_j, j \neq i$, which increases when R_j decreases. The capability of user i to decode the interference coming from interferer $j \neq i$ increases when R_j decreases.

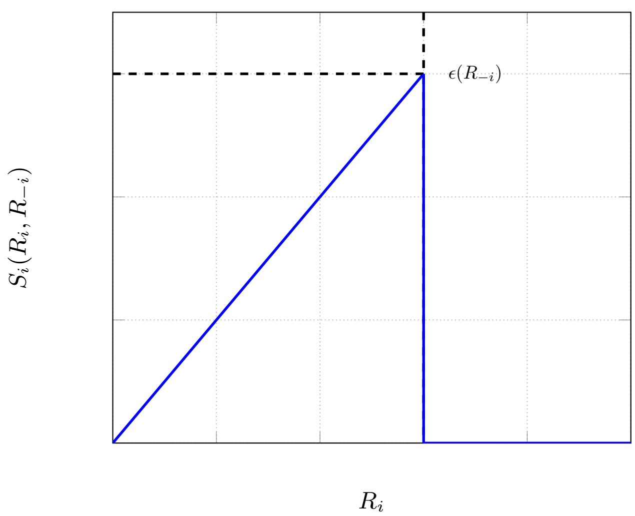

Figure 6.6: Obtained spectral efficiency after interference processing $S_i(R_i, R_{-i})$, for several values of R_i and fixed R_{-i}.

In the following, we define a criterion which immediately determines if user i is able to decode the interference signal coming from interferer $j \neq i$ in presence of other interference signals, as well as the maximal spectral efficiency after interference processing $\epsilon_i(R_{-i})$ for any user i, assuming the spectral efficiencies for each user are defined as R is then our concern in this section.

If UE i is unable to decode any interference coming from any interferer, it would not have any alternative but to treat the interferences as an additive noise of noise. When a user is unable to decode any interference from any of the $M-1$ other interferers, the spectral efficiency obtained after interference processing is minimal, and we denote it $R_n(j)$:

$$R_n(i) = \log_2 \left(1 + \frac{\gamma(i,i)}{1 + \sum_{\substack{k=1 \\ k \neq i}}^{M} \gamma(k,i)} \right) \tag{6.16}$$

This expression simply models that the maximal spectral efficiency for UE i, when processing interference as noise, is simply $\log_2(1+SINR_i)$, where $SINR_i$ is the SINR perceived at UE i. From now on, we consider that $R_n(i)$ is in fact the minimal spectral efficiency R_i that each UE i in the M-GIC can expect. It is a safe bet for user i, as an outage will never occur at this rate, but it does not take into account the possibility that some interference could be decoded, thus leading to a maximal spectral efficiency obtained after interference processing $\epsilon_i(R_{-i})$ potentially higher than $R_n(i)$.

From the point of view of UE i, UE i is able to decode the interference from interferer $j \neq i$, in presence of other interferers, if and only if the signal from AP j can be decoded in presence of the primary signal of UE i and other interferers signals from set $\alpha_i \subset \mathcal{M}^{-(i,j)}$, through its interfering link between AP j and UE i. The notation $\mathcal{M}$ denotes the interferers indexes $\mathcal{M} = \{1, ..., M\}$ and $\mathcal{M}^{-i}$ denotes the set $\mathcal{M}$ from which index i has been removed, i.e. $\mathcal{M}^{-i} = \{1, ..., i-1, i+1, ..., M\}$. We can then define the criterion which tells if UE i can decode the interference from interferer j, transmitting at spectral efficiency R_j, in presence of interfering signals in set α_i, as:

$$R_j \leq \log_2 \left(1 + \frac{\gamma(j,i)}{1 + \sum_{\substack{k \in \alpha_i \\ k \neq j}} \gamma(k,i)} \right) \Leftrightarrow \text{ interference can be decoded} \tag{6.17}$$

When an interferer j is decoded and removed, new possibilities of decodable interferers might appear. In that sense, the system will attempt to decode an additional interferer $k \in \alpha_i$, through the interfering link between AP k and UE i, in presence of interferers in set α_i', where α_i' is the updated previous set α_i, from which k has now been removed. We can then define, in Algorithm 2, the following iterative SIC procedure for any UE i, which returns the maximal spectral efficiency after interference processing for user i, $\epsilon_i(R_{-i})$.

Based on this algorithm, we observe that the optimal spectral efficiencies R_i^*

Data: Spectral efficiencies from other interferers R_{-i}, SNR/INR configurations

Result: The interference regimes user i, i.e. the set of decodable interferers β_i, the maximal spectral efficiency after interference processing $\epsilon_i(R_{-i})$, and the spectral efficiency R_i^* to be used by user i

Initialize the set of decodable interferers as $\beta_i = \{\}$.;

Initialize $\alpha_i = \mathcal{M}$.;

while D^{α_i} *is non-empty* **do**

Define D^{α_i}, the set of interferers $j \in \alpha_i$, that can be decoded by user i in presence of interferers in set α_i^{-j}. $j \in \alpha_i, j \neq i$ is in D^{α_i}, if and only if:

$$R_j \leq \log_2 \left(1 + \frac{\gamma(j,i)}{1 + \sum_{\substack{k \in \alpha_i^{-j} \\ k \neq j}} \gamma(k,i)} \right) \tag{6.18}$$

Update β_i by adding the interferers from set D^{α_i} to it.;

Update α_i by removing the interferers from set D^{α_i} from it.;

end

Once the set of all decodable interferers β_i is defined, we can compute the maximal spectral efficiency after interference processing $\epsilon_i(R_{-i})$, as:

$$\epsilon_i(R_{-i}) = \log_2 \left(1 + \frac{\gamma(i,i)}{1 + \sum_{j \in \alpha_i^{-i}} \gamma(j,i)} \right) \tag{6.19}$$

And the maximal spectral efficiency that user i can go for, without outage is then $\epsilon_i(R_{-i})$.

Algorithm 2: An iterative SIC procedure for defining the maximal spectral efficiency after interference processing to be used by any user i, in response to any spectral efficiencies combinations of the other users R_{-i}, is necessarily in the form of:

$$R_i^* = \log_2\left(1+\frac{\gamma(i,i)}{1+\sum_{j\in\alpha_i}\gamma(j,i)}\right) \tag{6.20}$$

With α_i being a subset of $\mathcal{M}^{-i}$.

When each user wishes to optimize its spectral efficiency, we reach a pure Nash Equilibrium (NE) for game (6.14), since it consists of a configuration where no user would deviate independently from its configuration $R_i^* = \epsilon_i(R_{-i})$ (a higher spectral efficiency would cause an outage, which would then return $S_i = 0$, while a lower R_i^* would lead to a lower S_i). We can prove the existence of pure NE, according to Proposition 6.4.

Proposition 6.4. *The game we have defined is*

- ***non-empty and metric:*** *this is guaranteed, because there are at least $M \geq 2$ players in the game, and the set of possible spectral efficiencies R_i for each user i is non-empty, but instead is a subset of a metric space.*
- ***compact:*** *the payoff functions S_i are bounded ($\forall i, S_i \in [0, \log_2(1+\gamma(i,i))]$). The maximal value is the point-to-point channel capacity for user i.*
- ***quasi-concave:*** *the payoff functions S_i are quasi-concave wrt R_j, for any j. This is intuitive and easily proven, by definition of the utility functions.*

According to [190, 191], there exist dominant strategies for each player i and for any configuration R_{-i} of the competing players, that we denoted $\epsilon_i(R_{-i})$. Also there exist at least one pure NE $R^* = (R_1^*, ..., R_M^*)$ if the game is also better-reply secure. Proving the better-reply secure condition in our game is not simple, but the existence of pure NE can still be proven under weaker conditions than the better-reply secure condition [192, 193]. We could also add that the payoff function only have finite values, since the optimal strategies are in the form of Equation (6.20) and the number of possible subsets α is finite, which helps proving the existence of at least a pure NE.

Defining the pure NE for a M-users discontinuous non-cooperative game verifying the conditions from Proposition 6.4, is known to be NP-Hard when the number of players M becomes greater than 2, more specifically it belongs to a class of problems called PPAD-Complete (where PPAD stands for 'Polynomial

Parity Arguments on Directed graphs') [194]. A possible procedure consists of transforming any M-users game into an equivalent 3-users game: Bubelis [195] proposes a reduction procedure which guarantees that for any solution of the equivalent 3-users game, a solution of the N-users game can be reconstructed using algebraic operations. Apart from the exhaustive algorithms, there exist a few algorithms to find pure NE, with reasonable computation times, but the task of detecting the NE of a finite strategic game remains today a challenging problem up-to-date [196]. Instead, we propose a simple iterative suboptimal approach for approaching a pure NE of the game, as detailed in Algorithm 3.

Data: SNR/INR configurations
Result: A possible Nash Equilibrium R^* for the game
Initialize the spectral efficiencies for all users R, where $\forall i$ as $R_i = R_n(i)$.;
while *A convergence criterion on R not reached* **do**
  Randomly pick a user i.;
  Compute maximal spectral efficiency $\epsilon_i(R_{-i})$ using Algorithm 2.;
  Update R_i as $R_i = \epsilon_i(R_{-i})$.;
end
When convergence is reached, the approached NE R^* returned by the iterative algorithm is $R^* = R$.

Algorithm 3: An iterative SIC procedure for defining the maximal spectral efficiency after interference processing.

Our proposed method for IC is able to take into account the possibility of decoding the interference at any user receiver side, when it does not affect the spectral efficiency of the interferer whose interference is being decoded. It should be noted that his approach does not guarantee a maximized total spectral efficiency for the M-users GIC. Instead it maximizes the individual performance of each interferer in the M-users GIC: it returns a stable NE configuration, in which every interferer has its individual spectral efficiency maximized.

6.4.3 Integer Linear Programming, NP-Hardness and Genetic Algorithms

Assuming we can compute the spectral efficiencies to be used by any interferer, in any M-GIC, returned by any matching configuration (m, u), our objective now consists of finding the optimal matching m^* and u^*, among all the possible matchings, such that the total performance of the system defined in Equation (6.13) is maximized. The definition of the matchings m and u, as well as the set of linear constraints in Equations (6.10), (6.11) and (6.12) may suggest that the optimization problem we are trying to solve belongs to the class of Integer Linear Programming (ILP) [106]. However, the objective function, and more specifically the individual spectral efficiencies after interference processing are not linear functions. For this reason we must consider different approaches, based on Non-Linear Programming [104, 105]. Due to the NP-Hardness of the Non-Linear Programming optimization problems, the classical branch and bound algorithms can only be considered for low dimension systems, as the problem becomes rapidly unsolvable when the system dimensions become large [197].

Instead, we consider in this thesis, suboptimal approaches, more specifically, evolutionary and genetic algorithms [108, 109, 110]. Those evolutionary algorithms are able to return suboptimal satisfying solutions in acceptable computation times and is able to return a suboptimal solution, taking into account both the non-linear objective functions and the linear inequalities. In this section, we provide more details about the Genetic algorithm we considered.

6.4.3.1 Preliminary on Genetic Algorithms

Initially introduced by [108, 109, 110], genetic algorithms are known as a family of search heuristics, inspired by evolution and often viewed as suboptimal function minimizers. An implementation of a genetic algorithm usually begins with a (random or given) population of $PopSize$ individuals, i.e. $PopSize$ independent realizations of the function to be minimized. In our case, an individual consists of an independent realizations of $x = [m, u]$. The population is then updated to form a new generation according to a generation process, which gives more 'chances to reproduce' to populations providing better solutions and explores new possible configurations for x based on the actual population through mutation, or crossover evolutions. The genetic algorithm then produces the next generation based on the previous one, according to the three following generation functions:

- *Elite offspring*: a given proportion $EliteFrac$, $0 < EliteFrac < PopSize$ of the population remains unchanged. More specifically, the $EliteFrac$ populations performing the best are identically reproduced in the next generation.

- *Crossover offspring*: a given proportion $0 < CrossFrac < PopSize-$

$EliteFrac$ of the population is updated according to a crossover process. The objective of a crossover is to consider two independent realizations of the current population and recombine them into one new children, by simply exchanging parts of their genome. As described in [108], several crossover techniques exist, such as the k-points crossover, the cut and splice or the uniform crossover. We considered a custom crossover function which is described in the next section.

- *Mutation offspring*: the remaining proportion of the population $MutFrac = PopSize - CrossFrac - EliteFrac$, is updated according to random mutations. This process updates an individual of the current population, by operating a random change in the considered individual. More details are also given in the next section.

The initial population function and the three generation functions can be designed such that both the initial population and its successive offsprings satisfy the linear constraints, which guarantee that x leads to matchings m and u that are realistically feasible. Details on the initial population generation is also provided in the next section. The genetic algorithm then runs until a convergence criterion is satisfied and the returned solution for $x = [m, u]$ is then the individual of the last generation performing the best. Usually, a good convergence criterion is a combination of the following terminating conditions:

- The maximal number of generations/computation time has been reached: this prevents the algorithm to run indefinitely.
- The population performance is reaching a plateau and successive generations are no longer producing better results: this allows the algorithm to stop when a convergence is observed.

Also, it is commonly admitted that the genetic algorithm should be run $N_i \geq 1$ independent times on a given problem. Among the N_i solutions returned by the N_i independent realizations of the genetic algorithm, the preferred solution is then the one returning the best performance. This enhance the capability of the system of finding a best-performing configuration x. An illustration of the genetic algorithm concept is provided in Figure 6.7.

6.4.3.2 The Initial Population Creation Function

In order to generate an initial population of $PopSize$ individuals, all of them satisfying the linear constraints, defined in Equations (6.10), (6.11) and (6.12),

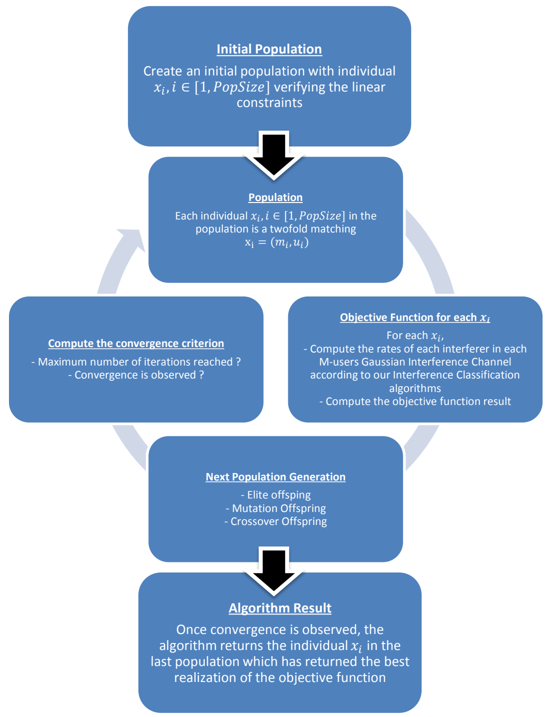

Figure 6.7: Overview of the Genetic Algorithm.

we propose to consider an initial random matching, which only make sure that the linear constraints, guaranteeing a set of feasible assignments, are satisfied. By enabling a random assignment procedure, we allow our algorithm to explore large sets of solutions, enhancing its capability of providing good-performing solutions. The pseudo-code related to the random assignment procedure is defined hereafter in Algorithm 4.

The algorithm can be slightly modified to prioritize the AP j and SRE combinations that provide a better SNR $\gamma(j, i)$. Instead, we let the algorithm perform uniformly random matchings, as it allows the genetic algorithm to explore a wider set of possible solutions, even though the initial populations might have an extremely poor performance.

Data: SNRs and INRs γ, τ
Result: Individual $x = [m, u]$
Initialize the set of unassigned users $U = \{1, ..., MN\}$;
Initialize the matching m and u as zeroes matrices;
while *U is non-empty* **do**
 Randomly pick an unassigned UE j in U;
 Assign UE j to a random available AP i and SRE k combination in the feasible set of combinations.;
 $\rightarrow m(i, j) = 1$ and $u(k, j) = 1$;
 Remove j from U.;
 Update the set of remaining possible assignments.;
end

Algorithm 4: Iterative Random Assignment Process for Creating an Initial Population

6.4.3.3 The Crossover Function

The crossover function exchanges and combines elements of two previous realizations. More specifically, if we consider two parent individuals of the current generation $x_1^p = [m_1^p, u_1^p]$ and $x_2^p = [m_2^p, u_2^p]$, the offspring for the next generation $x^o = [m^o, u^o]$ is then defined as follows. Let us first denote $\tilde{m}_1^p$, $\tilde{u}_1^p$, $\tilde{m}_2^p$ and $\tilde{u}_2^p$, defined as:

$$\tilde{m}_l^p(j) = i \text{ if UE } j \text{ is assigned to AP } i \text{ according to } m_l^p \tag{6.21}$$

$$\tilde{u}_l^p(j) = k \text{ if UE } j \text{ is assigned to SRE } k \text{ according to } u_l^p \tag{6.22}$$

The same way, we can define $\tilde{m}^o(j)$ and $\tilde{u}^o(j)$. The offspring matchings $x^o = [m^o, u^o]$ verifies:

$$\tilde{m}^o(j) = \tilde{m}_1^p(j) \text{ or } \tilde{m}^o(j) = \tilde{m}_2^p(j) \tag{6.23}$$

and

$$\tilde{u}^o(j) = \tilde{u}_1^p(j) \text{ or } \tilde{u}^o(j) = \tilde{u}_2^p(j) \tag{6.24}$$

The offspring AP-UE and SRE-UE matchings are then combinations of the parent population assignments. The following algorithm, described in Algorithm 5, is considered for crossover. This process guarantees that the offspring generations will respect the linear constraints, as did the previous generations and the initial population.

Data: Two parents $x_1^p = [m_1^p, u_1^p]$ and $x_2^p = [m_2^p, u_2^p]$
Result: Individual $x^o = [m^o, u^o]$
Initialize the set of unassigned AP and SREs $M = \{1, ..., N\}$ and $U = \{1, ..., N\}$.;
Initialize the matching m^o and u^o as zeroes matrices of size $M \times N$ and $N \times N$ respectively.;
Initialize $\tilde{m}^o(j)$ and $\tilde{u}^o(j)$ as zeroes vectors of length N.;
for *all UEs j picked in random order* **do**
  $[\tilde{m}^o(j), \tilde{u}^o(j)]$ = randomly select a combination from $\mathcal{C}(j)$.
end
Reconstruct $x^o = [m^o, u^o]$ based on $[\tilde{m}^o, \tilde{u}^o]$.;

Algorithm 5: Custom Crossover function

The combinations $\mathcal{C}(j)$ are defined at each iteration as:

$$\mathcal{C}(j) = \Big\{ (\tilde{m}^p_{l_1}(j), \tilde{m}^p_{l_2}(j)) \mid (l_1, l_2) \in \{1,2\}^2 \text{and} \nexists j_2 \neq j \in \{1, ..., N\} s.t. [\tilde{m}^p_{l_1}(j), \tilde{u}^p_{l_2}(j)] = [\tilde{m}^o(j_2), \tilde{u}^o(j_2)] \Big\} \tag{6.25}$$

6.4.3.4 The Mutation Function

In a similar way, we define the mutation function by simply considering isolated changes in the AP-UE or the SRE-UE assignations given by the current population $x^p = [m^p, u^p]$. For a given assignment, a mutation can happen randomly according to a given probability. The possible mutation changes are set so that the offspring $x^o = [m^o, u^o]$ satisfies the linear constraints. The mutation can happen on a given AP-UE assignment, on a SRE-UE assignment, or a combination of both AP and SRE assignments for one UE. Also, the mutation can involve an exchange of SRE/AP with another UE, as long as the offspring generated by this mutation satisfies the linear constraints. At least one mutation occurs and the probability of additional mutations happening on an individual is set low enough, so that the genetic search does not turn into a primitive random search.

6.4.3.5 Genetic Algorithm parameters

We sum up all the parameters we used in simulations for our genetic algorithm in Table 6.3, displayed hereafter. The parameters are similar to the ones used in the $M = 2$ scenario, detailed in Table 6.2, except for the number of AP M

which is equal to 10, and the number of unassigned UEs NM which is 50.

Parameter	Value
Population Size $PopSize$	20
Number of indep. real. of the Gen. Algo N_i	10
Fraction of elite offspring $EliteFrac$	2
Fraction of crossover offspring $CrossFrac$	10
Fraction of mutation offspring $MutFrac$	8

Table 6.3: Genetic Algorithm simulations parameters.

6.4.4 Numerical Simulations: $M > 2$ Case

In this section, we assume that the simulations parameters will be the same as those listed in section 6.3.2. We consider a system with 5 Macro-APs and 5 Femto-APs ($M = 10$ APs) and $NM = 50$ UEs. We consider 5 Interference Management Strategies (IMS) defined hereafter:

- **IMS 1**: orthogonalization is performed so that no UEs are interfering and each UE is assigned to its best AP.
- **IMS 2**: AP-UE assignments are randomly defined, so that each AP has N assigned UEs, but prioritize the UEs closest to the APs. Interferers matching is also random. Interferers have no other choice but to treat incoming interference as an additive source of noise.
- **IMS 3**: The AP-UE assignments and interferers matching are returned by our genetic algorithm, detailed in Section 6.4.3. No IC is considered, and interference is treated as an additive source of noise at each receiver side.
- **IMS 4**: The AP-UE assignments and interferers matching are returned by our genetic algorithm, detailed in Section 6.4.3. IC is considered, according to our study in Section 6.4.2.

We present in Figure 6.8 the histogram of the average spectral efficiency per UE, for each IMS.

As before, it appears that *IMS 1*, is spectrally inefficient. Considering *IMS 2* as a reference, it appears that optimizing the interferers matchings and the AP-UE assignments leads to notable performance improvements: allowing the system to select the best matching of interferers in the noisy regime (*IMS 3*)

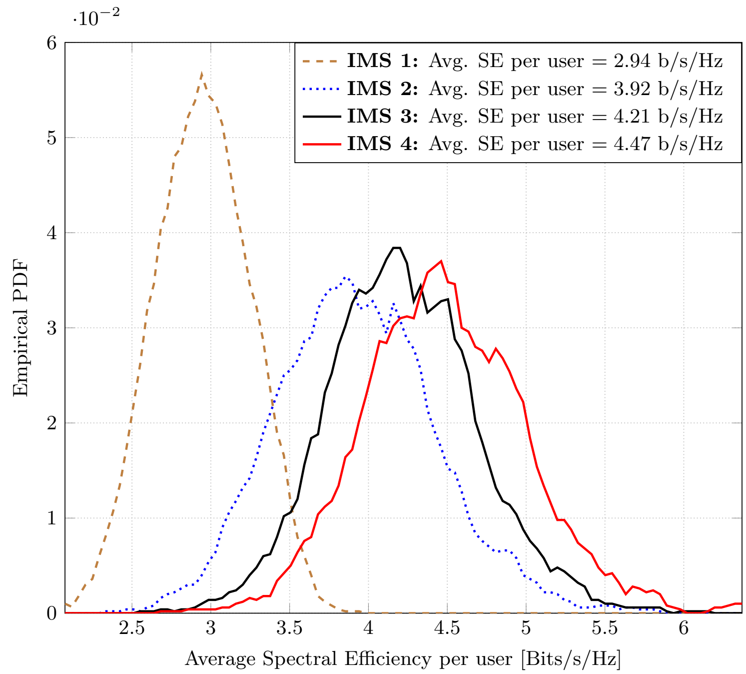

Figure 6.8: Histogram plot of the performances of each scenario, for $N_{MC} = 1000$ independent realizations ($M = 10$ interferers per group, $N = 5$ groups of interferers).

offers an average performance improvement of 7.4%, compared to *IMS 2*. The additional IC allows for an additional gain of 6.1 %.

6.5 Conclusions and Limits

In this chapter, we provided insights, suggesting that when IC is considered, the system must reconsider how it assigns its UEs to the available APs of the network: the best SNR AP is not always the best option anymore. We provided illustrative example demonstrating the potential gain offered by the concept of 'Virtual Handover', i.e. the possibility of reassigning the UEs to the APs, in scenarios where IC is considered. Later on, we start again the analysis of the proposed RRM optimization problem, conducted in Chapter 5 and investigate how the optimization is modified when considering Virtual Handover. When $M = 2$, an updated version of the previous '2-Regimes Interference Classification', is derived and takes into account both the IC and the AP-UE assignments.

The matching problem is then optimally solved using this '6-Configurations Interference Classification', from a graph theory point of view. In the $M > 2$ scenario, we first address the problem of IC in a M-GIC by proposing a suboptimal game-theoretic approach. A new IC method is then proposed, which is able to exploit SIC-based techniques, when it does not affect the performance of the other users in the system. Assuming we can compute the spectral efficiencies after interference processing, in any possible M-GIC, the objective of the matching problem is to define the optimal interferers matching m and AP-UE assignments u. The twofold matching problem appears to belong to a class of Non-Linear Programming problems, which is NP-Hard. We tackle the inherent mathematical complexity of the problem, by proposing a suboptimal genetic algorithm.

For both scenarios, we provide numerical simulations, that give insights on the performance gains offered respectively by IC, interferers matching and AP-UE reassignments. The presented works can be enhanced by taking into the following enhancements that we leave for future work:

- **Suboptimal interference classification:** We proposed a suboptimal IC for the M-GIC, when $M > 2$. As suggested before, defining the M-GIC is still an open question in literature. However, our presented work is modular: if a different IC technique was to be proposed for the M-GIC, it could be implement in our RRM approach. The twofold matching procedure based on the genetic algorithm we proposed,, remains valid and efficient, as long as we can define the total spectral efficiency after interference processing to be used in any M-GIC.

- **Genetic algorithm improvements:** The genetic algorithm we proposed is a suboptimal heuristic for solving the Non-Linear Programming problem. Additional investigation is necessary and could lead to a more efficient algorithm for solving the twofold matching. Also, it must be noted that the computational complexity of the proposed genetic algorithm poorly scales with the system dimensions M and N. For this reason, additional investigation could lead to a more efficient and low complexity algorithm, that could be used for real time applications.

- **Extension of the conducted analysis to different utility functions:** In the conducted study, we focused on maximizing the total performance, i.e. the total spectral efficiency after interference processing. As previously suggested in Section 5.6, different utilities functions could have been considered for optimization, which would require to start again the conducted analysis.

- **How will performance gains scale in a more realistic scenario ?** In numerical simulations, we considered arbitrary network/channel/power/SE models, for the sake of simplicity. Studying how the respective theoretical performance gains scale when a more realistic network model is considered, is obviously a matter of interest.

- **Robustness of the method to estimation errors:** it also intuitively appears that the proposed IC method might lack some robustness to estimation errors: in the proposed game-theoretic approach for IC in the M-GIC, the returned spectral efficiency is the maximal one before outage. Since the system is on the edge of outage, any estimation error, could immediately lead a user to an outage.

Chapter 7

Conclusions, Perspectives & Future Directions

7.1 Conclusions

Future wireless networks are expected to provide broadband access to a large number of mobile users, and satisfy the ever growing user data demand. Due to environmental and economic concerns, the 'green wireless networks' concept has been proposed as a way to enhance the energy efficiency of the network. In this thesis we investigate two approaches that are nowadays perceived as two promising techniques for future 5G green wireless networks: proactive delay-tolerant networking and interference classification based mitigation spectral efficiency optimization techniques. Herein, we summarize the contributions and conclusions of this work, according to the overall structure of the thesis. As a consequence, the remainder of the section is divided into two parts.

7.1.1 Conclusions from Part 1 - Chapter 3

In the first part of the manuscript, we exploited the well known latency vs. energy efficiency trade-off, and coupled it to the recent advances in terms of future network context predictions. To do so, we investigate a proactive delay-tolerant network, i.e. a scheduling problem on a given time window, for which we have elements of knowledge on the future transmission context. More specifically, we studied in Chapter 3, a single user proactive delay-tolerant scheduler, whose objective is to ensure a given data transmission before a given deadline at a minimal cumulated power cost. This first chapter served as a simple illustrative example which introduced the three main concepts that are used in the first half of this thesis: future knowledge, proactive resource allocation in delay-tolerant networks and optimization. Through this simple illustrative example, we provided answers to the following three fundamental questions related to delay-tolerant networks and future knowledge:

- **How can the system exploit some future knowledge?** A possible way for the system to exploit this future knowledge relies on exploiting the power-efficiency latency trade-off. We modeled a delay-tolerant transmitter, and consider a power control optimization problem, where the objective is to minimize the global power consumption required for completing a fixed transmission before a given deadline. The transmitter is cognitive and can adapt its transmission power to the present transmission context, in real time. The decision process for the optimal power strategy is then affected by the present state (time remaining before deadline, packet size remaining, etc.) but is also able to take into account some piece of future knowledge about the future transmission context. The conducted analysis reveals that the optimal power strategy can be obtained simply trough mathematical convex optimization, and more specifically, we proposed a backward dynamic programming algorithm which allows to compute the optimal strategy in any present configuration and for any kind of future knowledge available. After analyzing theoretically how this future knowledge is exploited by the system to compute the optimal power strategy, we provided numerical results demonstrating the average performance of the system, for several scenarios of future knowledge.

- **Does future knowledge offer significant performance gains?** The numerical simulations show that there is a significant gain between i) the zero knowledge scenario, which is the worst scenario of future knowledge, since the system does not know anything about the future transmission context, and thus is lower performance bound; and ii) the perfect knowledge scenario, which is the best scenario of future knowledge, since the system has perfect knowledge of the future at any time, and thus is the higher performance bound. Demonstrating that the gain was significant really matters: if the performance gap had not been significant enough, then looking for future knowledge, and providing it to the system, so that it can exploit it via scheduling and proactive resource allocation would not have made sense.

- **What kind of future knowledge is really useful to the system?** The conducted analysis shows that the system may greatly benefit from even partial future knowledge, and may almost reach the performance bound of the perfect knowledge scenario. Since acquiring a perfect knowledge at $t = 0$ seems unrealistic (even though ideal), we also investigated partial and statistical future knowledge schedulers. It turned out that a good statistical knowledge allows to approach remarkably the optimal performance bound. Also, acquiring a short-term knowledge, which is realistic, can also enhance the performance of the system. We also showed in Section 3.5.4 that the performance gap depends on the time variations of the channel realizations. More specifically, the performance gains depends on the capability of the system of discern good channel realizations from bad channels realizations and exploit them properly.

7.1.2 Conclusions from Part 1 - Chapter 4

In Chapter 4, we investigate the extension of the previous single user proactive delay-tolerant scheduling problem, to a multiuser scenario. A non-cooperative competition between users is considered, which leads to a multiuser non-cooperative stochastic game. The analysis of the game is conducted, and reveals the inherent mathematical complexity of solving such games in a stochastic configuration and with a large number of users N in the system. When faced with this mathematical complexity, three alternative approaches are classically considered in literature:

- An iterative time water-filling procedure can be considered, but requires no stochasticity and a number of users N small enough so that the computation time remains acceptable.

- A heuristic strategy, with low complexity can be considered, at the cost of suboptimality.

- A simplified version of the problem, e.g. constant channel scenarios, can be considered, or which the optimal power strategies can be explicitly computed.

In this thesis, we propose to exploit the recent advances in Mean Field Games theory to transition our initial game into an equivalent game with lower complexity, thus addressing the inherent mathematical complexity issue: the transition relies on the two following assumptions inherent to our system design: the number of users N is large enough to be considered infinite and the users in the system present symmetries (same rational behavior, same objective function and control sets, symmetric interactions, etc.). The conducted analysis of the equivalent Mean Field Game reveals that the optimal power strategy can be characterized with only two coupled PDEs (instead of N in the previous game), namely the HJB and the FPK equations. Moreover, the numerical simulations have revealed that the power strategies obtained through the Mean Field Game approach remarkably well the optimal power strategy when it can be computed (e.g. when there are no time variations on the channel realizations). The numerical simulations have also revealed a twofold gain, due to the Mean Field based scheduler capable of exploiting both the latency and the future knowledge. The observed gains are extremely significant, thus revealing the potential benefit offered by proactive delay-tolerant networks, for enhancing the energy efficiency of green wireless networks.

7.1.3 Shortcomings and Future Work - Part 1

The first part of the thesis provided insights on significant gains in terms of energy efficiency, offered by proactive delay-tolerant transmissions schemes. However, the presented works could be enhanced by taking into account several possible enhancements that we discuss hereafter:

- **More realistic channel model and practical proof of concept:** The system we considered is unrealistic on many points. For the sake of simplicity, we deliberately considered unrealistic arbitrary channel models, in order to observe specific behavior of both the Mean Field algorithm and the Mean Field strategy. The theoretical analysis provided insights on potential significant gains, that one could obtain by exploiting both the latency and the available future knowledge. A necessary extension of the presented work requires a realistic channel model, as it is necessary to observe if the potential theoretical gains will scale when considering realistic channel models. The channel model we considered in the game definition, consisted of an auto-regressive process of order 1, with both a deterministic part (used to model an accurate prediction) and a stochastic part (used to model uncertainty). It should be noted that such models have been widely used in Mean Field Games mathematical theory, but they suffer from a flaw, when we use them to model channels in the telecommunication field: due to the stochastic part, there is a non-zero probability that any channel $h(t)$ might go to infinity, when the time t tends to infinity as well. That is the reason why such an auto-regressive model can be questioned, when used to model channel evolution. However, for the moment, these models are the only ones for which we have theoretical results in Mean Field theory. Several ongoing works have however tried to extend the Mean Field Theory to different evolution models, as listed in [170]. The objective for future work could consist of realizing a practical proof of concept in a realistic practical scenario, in order to understand how the theoretical observed gain will scale when transposed into a more realistic practical context.

- **Pertinence of the MFG assumptions:** When transitioning to the Mean Field Game, we made an assumption on the users indistinguishability. This indistinguishability property allowed to regroup $N-1$ users into a mean field, thus simplifying greatly the system, but it also implies that the primary channels used for transmission in each AP-UE pair, have the same dynamics. Such a strong hypothesis can be questioned, especially for a practical proof of concept. To address this issue, future theoretical work may include different classes of users, in order to model different behaviors of users (e.g. different types of mobilities), different types of APs, etc. When $M, M < N$ different classes of users are considered, the Mean Field Equilibrium analysis becomes more complex, as we must solve M HJB equations, in order to obtain the optimal power strategy to be used by every user, depending on the class it belongs to, as introduced by Nash in 1951 [171], each equation corresponding to the M different classes. In our analysis, we assumed only one class of users, thus leading to only one HJB equation, used for computing the optimal strategy to be used by every user in the system. an extension of the presented work, in heterogeneous networks for example, might require to consider classes of users. The extreme case with N classes of one user, leads to a set of N HJB equations, which is strictly equivalent to the analysis of the multiuser non-cooperative stochastic game.

- **Acquiring future knowledge and cost of learning:** In the conducted analysis, we have not questioned how the elements of future knowledge could be obtained. In particular, we must discuss the 'cost of learning', namely the equivalent power cost required, in order to acquire some elements of future knowledge. Investigating this 'cost of knowledge' is a difficult task and still an open question in research at the moment: at the best of our knowledge, there are only a few limited works that are trying to explicit this 'cost of learning'. A few ideas could be found in here [137], even though it is not directly related to wireless networks. More works however focus on defining the 'cost of feedback' [138], namely the cost one has to pay to transmit a piece of information from a central unit in charge of establishing predictions to the AP that needs it. It is a matter of importance, since we need to confront this 'cost of learning' to the potential performance gain that the system could benefit from the acquired future knowledge.

- **Different utility function and sleep mode:** In the presented work, we have again assumed that the objective was to minimize a utility function consisting of the total transmission power exclusively. We could enhance the power consumption model by considering, for example, a more complete power consumption model, which takes into account the operating and primary costs of an AP, as suggested in [8]. If such a model was considered, less importance would be given to the transmission power costs and we would probably consider scenarios where the AP can be turned into idle mode, when unused for transmission, which is also a promising feature for power efficiency [13, 14, 15]. It could lead to a new class of strategies, able to take into account sleeping modes, whose investigation could be of interest.

- **Limited knowledge and distributed approach:** In this chapter, we assumed a perfect knowledge at of the system parameters and evolution of every pair of AP-UE in the system, at each pair of AP-UE. A distributed approach, with limited knowledge at each AP-UE pair can also be investigated, with an inspiring example in [81].

- **High performance heuristic strategies:** Our choice of heuristic strategies was simple. We modeled a full-power strategy, as a simple way to define a strategy that is unable to take into account neither the latency nor the channel evolution. Future work will also investigate more sophisticated heuristic strategies, that are more efficient, in terms of energy efficiency and can be used for real-time applications, thanks to its low complexity.

7.1.4 Conclusions from Part 2 - Chapter 5

In the second part of the thesis, we addressed a dual problem to the power control proactive delay-tolerant scheduling problem, i.e. we proposed to enhance the spectral efficiency of the network under a constraint of a constant short-term power configuration. To do so, we proposed to exploit an underexploited property of the network, namely the inherent properties of interference, highlighted by recent works from Etkin and Tse on interference classification [85]: these works revealed that treating interference as noise is not always the best option. To do so, we proposed to investigate the optimization problem of maximizing the total spectral efficiency after interference processing in Gaussian Interference Channels, by selecting the most appropriate interference regime (i.e. interference mitigation technique at each receiver side) and the spectral efficiencies to be used by each pair of AP-UE. The analysis was first conducted in a 2-users Gaussian Interference Channel and revealed a low complexity interference classification, with only two dominant regimes: the noisy regime and the SIC-based regime. Based on this observation, it appeared that the system may not find any interest in avoiding the interference, as proposed in the reference orthogonalization-based strategies. We left in this chapter, the investigation of the M-users Gaussian Interference Channel with $M > 2$ users, as it leads to a more complex case study, with a large number of cases including all possible combinations of Successive Interference Cancellation. It should be noted that the optimal interference classification in M-users Gaussian Interference Channels is still an open question in research.

The proposed interference classification can then be exploited in a scenario with multiple interferers per AP. The extension of the previous optimization to a scenario with multiple interferers per AP led to a matching problem, whose objective was to form the optimal groups of interferers, with exactly one interferer from each AP in each group. In each group of interferers and in the $M = 2$ case, the interference classification could be considered in each M-users Gaussian Interference Channel and was based on our previous interference classification results. The objective consisted of finding the optimal couples between two coalitions of N interferers. We showed that the problem can be optimally solved using the Kuhn-Munkres algorithm. However, when the number of AP

M becomes greater than 2, no interference classification could be simply considered. Nevertheless, we proposed to investigate the optimization problem of interferers matching and proposed to exploit a heuristic genetic algorithm to tackle the inherent mathematical complexity of the Multidimensional Assignment Problem in $M > 2$ dimensions. Numerical simulations provided interesting potential performance gains for the system, which is twofold: a first gain is offered by interference classification and can be enhanced by an interferers matching approach.

7.1.5 Conclusions from Part 2 - Chapter 6

Chapter 6 begins with an observation: when interference classification is considered, the classical way of assigning UEs to APs is no longer valid. As a matter of fact, it appeared that the AP providing the best SNR, which was the best option when interference was treated as noise exclusively, is not necessarily the best option anymore, when interference classification is considered. For this reason, we proposed to start again the previously conducted analysis, by considering that the UEs were no longer assigned to APs, but that the system had to form coalitions of N interferers, assigned to each AP, in addition to the previous interference classification and matching.

We first derived an updated interference classification and AP-UE assignment in 2-users Gaussian Interference Channels, that immediately tells, for any pair of APs and UEs, the best AP-UE assignments, as well as the best interference regimes and spectral efficiencies, so that the total spectral efficiency after interference processing was maximized. We proposed to exploit the new-built interference classification to address the problem of interferers matching in the $M = 2$ case. We showed that the matching problem can be optimally solved, by using a graph theory based approach, which relies on the Edmonds algorithm.

In this chapter, we also investigated the $M > 2$ case. In the previous analysis, two difficulties arose. First, there is no easy interference classification for M-users Gaussian Interference Channels. To address this issue, we proposed a game-theoretic interference classification approach, which is based on the two dominant interference regimes, namely the noisy and SIC-based interference regimes. The proposed interference classification is suboptimal but has a low complexity and guarantees a minimal spectral efficiency for each user, which corresponds to the spectral efficiency obtained when all interferers are treated as noise. The proposed interference classification is then exploited in a twofold matching problem, which consists of defining both the AP-UE assignments and the matching of interferers. The optimization problem turns out to be a Non-Linear Programming problem, which is known to be NP-Hard. To tackle the mathematical complexity, we propose a Genetic Algorithm, that can be used to find a satisfying matching, exploiting the previous interference classification. Numerical simulations highlight potential performance gains, in terms of total spectral efficiency, due to interference classification, that could be enhanced by both the AP-UE reassignments and the interferers matching.

7.1.6 Shortcomings and Future Work - Part 2

The second part of the thesis provided insights on significant gains in terms of spectral efficiency at fixed power configurations, offered by our interference classification based RRM techniques. However, the presented works could be enhanced by taking into account several possible enhancements that we discuss hereafter:

- **Suboptimal interference classification in M-GIC:** We proposed a suboptimal interference classification for the M-users Gaussian Interference Channel, when $M > 2$. As suggested before, defining the M-users Gaussian Interference Channel is still an open question in literature. However, our presented work is modular: if a different interference classification technique was to be proposed for the M-users Gaussian Interference Channel, it could be implement in our RRM approach. The twofold matching procedure based on the genetic algorithm we proposed,, remains valid and efficient, as long as we can define the total spectral efficiency after interference processing to be used in any M-users Gaussian Interference Channel.

- **Genetic algorithm improvements:** The genetic algorithm we proposed is a suboptimal heuristic for solving the Non-Linear Programming problem. Additional investigation is necessary and could lead to a more efficient algorithm for solving the twofold matching. Also, it must be noted that the computational complexity of the proposed genetic algorithm poorly scales with the system dimensions M and N. For this reason, additional investigation could lead to a more efficient and low complexity algorithm, that could be used for real time applications.

- **Extension of the conducted analysis to different utility functions:** The conducted analysis has the objective of maximizing the total performance of the system, i.e. the total spectral efficiency after interference processing. Several other utility functions could have been considered [86], such as: maximizing the minimal spectral efficiency offered to each user after interference processing, maximizing a weighted spectral efficiencies sum, maximizing the total spectral efficiency of a specific set of users, etc. Modifying the utility function would require to conduct the interference classification analysis and matching problems again, probably leading to different results.

- **How will performance gains scale in a more realistic scenario ?** In numerical simulations, we considered arbitrary network/channel/power/SE models, for the sake of simplicity. Studying how the respective theoretical performance gains scale when a more realistic network model is considered, is obviously a matter of interest. It also intuitively appears that the proposed interference classification method might lack some robustness to estimation errors: in the proposed game-theoretic approach for interference classification in the M-users Gaussian Interference Channel, the returned spectral efficiency is the maximal one before outage. Since the system is on the edge of outage, any estimation error, could immediately lead a user to an outage. This kind of issue must be take into account, especially if a practical proof-of-concept was to be considered.

Chapter 8

Appendices

8.1 Proof of Proposition 3.1

The objective is to find the optimal power strategy p^*_{om} to the optimization problem (3.2):

$$p^*_{om} = (p^*_{om}(1), p^*_{om}(2), ...p^*_{om}(T)) = \arg\min_p \left[\sum_{k=1}^{k=T} p(k)\right] = f(p)$$

$$\text{s.t., } Q(T) = Q(0) - \sum_{k=1}^{k=T} B \log_2 \Big(1 + h_r(k)p(k)\Big)\Delta t = 0$$

And $\forall t, i \in \{1, ..., T\}, i > t$, $D_i^t(h)$ is the prediction made by the system about $h_r(i)$ at the beginning of TS t.

Using the Karesh-Kuhn-Tucker conditions immediately leads to:

- Stationarity: $-\nabla f(p^*_{om}) = -\sum_{t=1}^{T} \alpha_t \nabla p(t) + \beta \nabla \Big(Q(0) - \sum_{k=1}^{k=T} B \log_2 \Big(1 + h_r(k)p^*_{om}(k)\Big)\Delta t\Big)$
- Primal Feasibility 1: $\forall t, p^*_{om}(t) \geq 0$
- Primal Feasibility 2: $Q(0) - \sum_{k=1}^{k=T} B \log_2 \Big(1 + h_r(k)p^*_{om}(k)\Big)\Delta t = 0$
- Dual feasibility: $\forall t, \alpha_t \geq 0$
- Complementary Slackness: $\forall t, \alpha_t p^*_{om}(t) = 0$

It immediately follows from the complementary slackness and primal feasibility 1 conditions that $\forall t, \alpha_t = 0$ or $p^*_{om}(t) = 0$. The stationarity condition can then be rewritten as:

$$-\nabla f(p^*_{om}) = \beta\nabla\left(Q(0) - \sum_{k=1}^{k=T} B\log_2\Big(1 + h_r(k)p^*_{om}(k)\Big)\Delta t\right) \tag{8.1}$$

If we ignore for now that $\forall t, \alpha_t = 0$ or $p^*_{om}(t) = 0$, the stationarity condition rewrites:

$$\Leftrightarrow \forall t, 1 = \beta\frac{Bh_r(t)}{log(2)}\frac{1}{(p^*_{om}(t)h_r(t)+1)} \tag{8.2}$$

$$\Leftrightarrow \forall t, p^*_{om}(t) = \frac{B\beta}{log(2)} - \frac{1}{h_r(t)} \tag{8.3}$$

In order to take into account that $\forall t, \alpha_t = 0$ or $p^*_{om}(t) = 0$, it is sufficient to pose:

$$\Leftrightarrow \forall t, p^*_{om}(t) = \left(\mu - \frac{1}{h_r(t)}\right)^+ = max(0, \mu - \frac{1}{h_r(t)}) \tag{8.4}$$

Where $\mu = \frac{B\beta}{log(2)}$.

The $\forall t, \alpha_t = 0$ or $p^*_{om}(t) = 0$ condition is verified by forcing $\alpha_t = 0$, when $p^*_{om}(t) > 0$. The only constraint remaining is then the primal feasibility 2 constraint:

$$Q(0) - \sum_{k=1}^{k=T} B\log_2\Big(1 + h_r(k)p^*_{om}(k)\Big)\Delta t = 0 \tag{8.5}$$

Which rewrites

$$\frac{Q(0)}{B\Delta t} = \sum_{t=1}^{T}\Big(log_2(\mu h_r(t))\Big)^+ \tag{8.6}$$

Proving that there exist a unique μ satisfying equation (8.6) is simple, due to the convexity of the equation. Computing a closed-form formula for μ is impossible, as it depends on the number of TS $\mathcal{N}(Q(0), h)$, for which we have $log_2(\mu h_r(t)) \geq 0$, i.e. $\mu \geq \frac{1}{h_r(t)}$. $\mathcal{N}(Q(0), h)$ is then defined as:

$$\mathcal{N}(Q(0), h) = card\,\{p_{om}(t) > 0 \mid t \in \{1, ...T\}, Q(0), h\} \tag{8.7}$$

However, the numerical value of μ can be approached via dichotomic search.

8.2 Proof of Proposition 3.2

Let us first recall the previous results. $\forall t \in \{1, ...T\}$, the optimization problem leads to the following time water-filling solution:

$$\forall k \in \{t, ..., T\},\, p(k) = \begin{cases} max(0, \mu - \frac{1}{h_r(t)}) & \text{if } k = t \\ max(0, \mu - \frac{1}{\epsilon}) & \text{else.} \end{cases} \tag{8.8}$$

Where μ is the unique water-level satisfying:

$$Q(t-1) = \sum_{k=t}^{T} B \log\left(1 + h_r(k)p(k)\right)\Delta t \tag{8.9}$$

Let us now define the condition on which the system will complete its transmission on time slot t leaving the $(T-t)$ remaining time slots unexploited. If the system completes the transmission on time slot t, then the water level used for $p(k)$, solving the optimization problem, $\mu(t) = \frac{2^{\frac{Q(t-1)}{B\Delta t}}}{h_r(t)}$. Also, the water level μ has to be greater than $\frac{1}{\epsilon}$, so that the system will not plan on using the $(T-t)$ future time slots. This implies that $h_r(t) \geq 2^{\frac{Q(t-1)}{B\Delta t}}\epsilon$.

If the system does not complete the transmission on the first time slot t, then the water level used for defining p_t, $\mu(t)$ becomes:

$$\mu(t) = \sigma_n^2 \left(2^{\frac{Q(t-1)}{B\Delta t}} \frac{1}{(\epsilon)^{T-t} h_r(t)} \right) \tag{8.10}$$

Reinjecting the new expression of $\mu(t)$ in

$$Q(t) = Q(t-1) - B\log_2\left(p(t)h_r(t)\right)\Delta t \tag{8.11}$$

leads immediately to

$$Q(t) = \frac{T-t}{T-t+1}\left[Q(t-1) - B\log_2\left(\frac{h_r(t)}{\epsilon}\right)\Delta t \right]^+ \tag{8.12}$$

Using a recurrence proof scheme, we can compute, $\forall t \in \{0, ..., T\}$, the value of $Q(t)$ and relate it to $Q(0)$, according to

$$Q(t) = \left(\frac{T-t}{T} Q(0) - \sum_{i=1}^{t} \frac{T-t}{t-i+1} B\log_2\left(\frac{h_r(i)}{\epsilon}\right)\Delta t \right)^+ \tag{8.13}$$

The power strategy $p^*_{zk}(t)$ is then easily related to the amount of data transfered during time slot t, which is given by $Q(t-1)-Q(t)$, as:

$$Q(t-1)-Q(t) = B\log_2\left(1+p^*_{zk}(t)h_r(t)\right)\Delta t \tag{8.14}$$

And hence,

$$p^*_{zk}(t) = \left(2^{\frac{(Q(t-1)-Q(t))}{B\Delta t}}-1\right)\frac{1}{h_r(t)} \tag{8.15}$$

8.3 Discussing Proposition 4.4

In order to find the expression of the powers at time t, as a function of $v^*_i, i\in\mathcal{N}$, we muist solve a set of N equations, namely:

$$\forall i\in\mathcal{N}, 1+\frac{\partial w_i(t,X,p)}{\partial p_i(t)}\partial_{Q_i}v^*_i+\sum_{j\neq i}\frac{\partial w_j(t,X,p)}{\partial p_i(t)}\partial_{Q_j}v^*_i=0 \tag{8.16}$$

Which rewrites

$$1+\frac{h_{ii}(t)B}{\log(2)\left(p_i(t)h_ii(t)+\sigma_n^2+\frac{1}{N-1}\sum\limits_{j\neq i}p_j(t)h_{ji}(t)\right)}\partial_{Q_i}v^*_i$$
$$+\sum_{j\neq i}\frac{h_{jj}(t)p_j(t)B}{log(2)\left(\sigma_n^2+\frac{1}{N-1}\sum\limits_{k\neq j}h_{kj}(t)p_k(t)\right)}\frac{1}{\left(\sigma_n^2+\frac{1}{N-1}\sum\limits_{k\neq j}h_{kj}(t)p_k(t)+h_{jj}(t)p_j(t)\right)}\partial_{Q_j}v^*_i=0 \tag{8.17}$$

The set of N equations can be rewritten as N polynomial equations in $p_i(t), i\in\mathcal{N}$, with order $2N-1$.

8.4 Proof of Proposition 5.1

Our objective is first to show, that, for any SNR/INR configuration, the regimes (1,2) and (2,1) are outperformed by either (1,1), (2,2), (1,3) or (3,1), when focusing on the total spectral efficiency. In the following, we denote by L, the following term:

$$L=\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)+\log_2\left(1+\frac{\gamma_2}{1+\delta_2}\right) \tag{8.18}$$

We first demonstrate that $(2,2)\triangleright(1,2)$ is equivalent to $L<\log_2(1+\gamma_2)$.

Indeed, the maximal achievable spectral efficiency for the regime (1,2) is

$$R(1,2) = \frac{1}{2}L + \frac{1}{2}\log_2(1+\gamma_1)$$

The maximal achievable spectral efficiency for the regime (2,2) is

$$R(2,2) = \frac{1}{2}\log_2(1+\gamma_1) + \frac{1}{2}\log_2(1+\gamma_2)$$

Then, $(2,2) \rhd (1,2)$ is equivalent to $R(1,2) \leq R(2,2)$, which immediately leads to $L < \log_2(1+\gamma_2)$.

The same way, we can demonstrate that:

- $(2,2) \rhd (2,1)$ is equivalent to $L \leq \log_2(1+\gamma_1)$.
- $(1,1) \rhd (1,2)$ is equivalent to $L > \log_2(1+\gamma_1)$.
- $(1,1) \rhd (2,1)$ is equivalent to $L \geq \log_2(1+\gamma_1)$

From the previous four propositions, we listed in Table 8.1, the best regime among (1,1), (1,2), (2,1) and (2,2) for every possible configuration.

Table 8.1: Summary of best regime for each configuration

	$L > \log_2(1+\gamma_1)$	$L < \log_2(1+\gamma_1)$
$L > \log_2(1+\gamma_2)$	$(1,1)$	$(1,2)$
$L < \log_2(1+\gamma_2)$	$(2,1)$	$(2,2)$

From the previous table, we can state that (1,2) and (2,1) are outperformed by either (1,1) or (2,2), except when $\log_2(1+\gamma_2) > L > \log_2(1+\gamma_1)$ or $\log_2(1+\gamma_2) < L < \log_2(1+\gamma_1)$. In the following, we focus on the scenario, consisting of $\log_2(1+\gamma_2) > L > \log_2(1+\gamma_1)$, which leads to the 'a priori' best configuration (1,2). It turns out that when $\log_2(1+\gamma_1) > L > \log_2(1+\gamma_2)$, $(3,1) \rhd (1,2)$.

Indeed, the maximal achievable spectral efficiency for the regime (3,1) is

$$R(3,1) = \log_2(1+\gamma_1) + \log_2\left(1 + \min\left[\frac{\gamma_2}{1+\delta_2}, \frac{\delta_1}{1+\gamma_1}\right]\right)$$

We compute

$$R(3,1) - R(1,2) = \frac{1}{2}\left[\log_2(1+\gamma_1) - L\right]$$

$$+\log_2\left(1+\min\left[\frac{\gamma_2}{1+\delta_2},\frac{\delta_1}{1+\gamma_1}\right]\right)$$

Since $\log_1(1+\gamma_2) > L$, necessarily $R(3,1)-R(1,2) > 0$, which means that $(3,1)\triangleright(1,2)$.

The same way, when $\log_2(1+\gamma_1) < L < \log_2(1+\gamma_2)$, we have $(1,3)\triangleright(2,1)$. And we can finally state that, for any configuration, (1,2) and (2,1) are outperformed either by (1,1), (2,2), (3,1) or (1,3).

Our objective is now to show, that, for any SNR/INR configuration, the regimes (3,2) and (2,3) are outperformed, in terms of total spectral efficiency, respectively by (3,1) and (1,3). Since the regimes (2,3) and (3,2) are symmetric, we focus only on the regime (2,3).

The maximal achievable spectral efficiency for the regime (2,3) is

$$R(2,3)=\log_2(1+\gamma_2)+\min\left[\frac{1}{2}\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right),\log_2\left(\frac{\delta_2}{1+\gamma_2}\right)\right]$$

The maximal achievable spectral efficiency for the regime (3,1) is

$$R(1,3)=\log_2(1+\gamma_2)+\min\left[\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right),\log_2\left(\frac{\delta_2}{1+\gamma_2}\right)\right]$$

Since $\min\left[\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right),\log_2\left(\frac{\delta_2}{1+\gamma_2}\right)\right] \geq \min\left[\frac{1}{2}\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right),\log_2\left(\frac{\delta_2}{1+\gamma_2}\right)\right]$, it comes immediately that $R(2,3)\leq R(1,3)$, which means $(1,3)\triangleright(2,3)$.

Finally, we demonstrate that (2,2) is always outperformed by either (1,3) or (3,1). For example, the maximal achievable spectral efficiency for the regime (2,2) is

$$R(2,2)=\frac{1}{2}\log_2(1+\gamma_1)+\frac{1}{2}\log_2(1+\gamma_2)$$

The maximal achievable spectral efficiency for the regime (3,1) is

$$R(3,1)=\log_2(1+\gamma_1)+\min\left[\log_2(1+\gamma_2),\log_2\left(\frac{\delta_1}{1+\gamma_1}\right)\right]$$

We can state that, for any configuration

$$R(3,1)-R(2,2)=\frac{1}{2}\left[\log_2(1+\gamma_1)-\log_2(1+\gamma_2)\right]+\min\left[\log_2(1+\gamma_2),\log_2\left(\frac{\delta_1}{1+\gamma_1}\right)\right]$$

A sufficient condition to $R(3,1)-R(2,2) \geq 0$, i.e $(3,1) \triangleright (2,2)$, is $\gamma_1 > \gamma_2$.

Similarily, we can show that a sufficient condition to $(1,3) \triangleright (2,2)$ is $\gamma_1 < \gamma_2$. From both previous statements, we can conclude that, for any configuration, $(3,1) \triangleright (2,2)$ or $(1,3) \triangleright (2,2)$.

8.5 Proof of Proposition 5.2

Let us first recall the performance of the three considered regimes, given by:

$$R(1,1) = \log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)+\log_2\left(1+\frac{\gamma_2}{1+\delta_2}\right)$$

$$R(1,3) = \log_2\left(1+\min\left[\frac{\gamma_1}{1+\delta_1}, \frac{\delta_2}{1+\gamma_2}\right]\right)+\log_2\left(1+\gamma_2\right)$$

$$R(3,1) = \log_2\left(1+\min\left[\frac{\gamma_2}{1+\delta_2}, \frac{\delta_1}{1+\gamma_1}\right]\right)+\log_2\left(1+\gamma_1\right)$$

Let us first consider C defined by

$$\begin{aligned} C = \log_2&\left(1+\min\left[\frac{\gamma_1}{1+\delta_1}, \frac{\delta_2}{1+\gamma_2}\right]\right)+\log_2\left(1+\gamma_2\right) \\ &-\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)-\log_2\left(1+\frac{\gamma_2}{1+\delta_2}\right) \end{aligned}$$

Firstly, by definition of the $\triangleright$ operator, we state that $(1,3) \triangleright (1,1) \Leftrightarrow C \geq 0$. One can easily verify that $\frac{\delta_2}{1+\gamma_2} \geq \frac{\gamma_1}{1+\delta_1}$ is a sufficient condition for $(1,3) \triangleright (1,1)$. From this, we deduce that $\frac{\gamma_1}{1+\delta 1} \geq \frac{\delta_2}{1+\gamma_2}$ is a necessary, yet not sufficient, condition for $(1,1) \triangleright (1,3)$.

Assuming $\frac{\delta_2}{1+\gamma_2} \leq \frac{\gamma_1}{1+\delta_1}$, C rewrites:

$$\begin{aligned} C' = \log_2&\left(1+\frac{\delta_2}{1+\gamma_2}\right)+\log_2\left(1+\gamma_2\right) \\ -\log_2&\left(1+\frac{\gamma_1}{1+\delta_1}\right)-\log_2\left(1+\frac{\gamma_2}{1+\delta_2}\right) \end{aligned}$$

Which rewrites:

$$C' = \log_2(1+\delta_2)-\log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)$$

In this configuration, one can verify that $C' \leq 0$ is equivalent to $\frac{\gamma_1}{1+\delta_1} > \delta_2$. In the end, we get that $(1,1) \triangleright (1,3) \Leftrightarrow \left[\frac{\gamma_1}{1+\delta_1} \geq \delta_2 \text{ and } \frac{\gamma_1}{1+\delta_1} \geq \frac{\delta_2}{1+\gamma_2}\right]$, which immediately leads to $(1,1) \triangleright (1,3) \Leftrightarrow \frac{\gamma_1}{1+\delta_1} \geq \delta_2$.

The same way, we can prove that $(1,1) \triangleright (3,1) \Leftrightarrow \gamma_2 \geq \delta_1(1+\delta_2)$ holds.

8.6 Proof of Proposition 5.3

Let us first recall the performances for each regime

$$R(1,3) = \log_2\left(1+\min\left[\frac{\gamma_1}{1+\delta_1}, \frac{\delta_2}{1+\gamma_2}\right]\right)+\log_2(1+\gamma_2)$$

$$R(3,1) = \log_2\left(1+\min\left[\frac{\gamma_2}{1+\delta_2}, \frac{\delta_1}{1+\gamma_1}\right]\right)+\log_2(1+\gamma_1)$$

$$\begin{aligned} R(3,3) &= \log_2\left(1+\min\left[\gamma_1, \tfrac{\delta_2}{1+\gamma_2}\right]\right) \\ &+ \log_2\left(1+\min\left[\gamma_2, \tfrac{\delta_1}{1+\gamma_1}\right]\right) \end{aligned}$$

We focus on defining a criterion based on $\gamma_1, \gamma_2, \delta_1, \delta_2$ for $(1,3) \triangleright (3,3)$. In a symmetrical way, one can deduce a criterion for $(3,1) \triangleright (3,3)$, based on $(1,3) \triangleright (3,3)$. Let us also define C_{13-33} as:

$$C = \log_2(1+\gamma_2)+\log_2\left(1+\min\left[\frac{\gamma_1}{1+\delta_1}, \frac{\delta_2}{1+\gamma_2}\right]\right)$$
$$-\log_2\left(1+\min\left[\gamma_1, \frac{\delta_2}{1+\gamma_2}\right]\right)-\log_2\left(1+\min\left[\gamma_2, \frac{\delta_1}{1+\gamma_1}\right]\right)$$

And $(1,3) \triangleright (3,3) \Leftrightarrow C \geq 0$.

Proposition 8.1. *A sufficient condition for* $(1,3) \triangleright (3,3)$ *is* $\gamma_2 \geq \delta_1$. *A sufficient condition for* $(3,1) \triangleright (3,3)$ *is* $\gamma_1 \geq \delta_2$.

Proof. If $\gamma_2 \leq \delta_1 \leq \frac{\delta_1}{1+\gamma_1}$, C rewrites

$$\begin{aligned} C &= \log_2(1+\gamma_2)+\log_2\left(1+\min\left[\frac{\gamma_1}{1+\delta_1}, \frac{\delta_2}{1+\gamma_2}\right]\right) \\ &- \log_2\left(1+\min\left[\gamma_1, \frac{\delta_2}{1+\gamma_2}\right]\right)-\log_2\left(1+\frac{\delta_1}{1+\gamma_1}\right) \end{aligned}$$

At this point, we may distinguish 3 cases.

1. $\gamma_1 \leq \frac{\delta_2}{1+\gamma_2}$

2. $\gamma_1 \geq \frac{\gamma_1}{1+\delta_1} \geq \frac{\delta_2}{1+\gamma_2}$
3. $\gamma_1 \geq \frac{\delta_2}{1+\gamma_2} \geq \frac{\gamma_1}{1+\delta_1}$

- When $\gamma_1 \leq \frac{\delta_2}{1+\gamma_2}$, we get $C \geq 0 \Leftrightarrow \gamma_2 \geq \delta 1$, which is true, by definition.
- When $\gamma_1 \geq \frac{\gamma_1}{1+\delta_1} \geq \frac{\delta_2}{1+\gamma_2}$, we get $C \geq 0 \Leftrightarrow \gamma_2 \geq \frac{\delta_1}{1+\gamma_1}$, which is also true, by definition.
- Finally, when $\gamma_1 \geq \frac{\delta_2}{1+\gamma_2} \geq \frac{\gamma_1}{1+\delta_1}$, a sufficient condition for $C \geq 0$ is $\frac{\delta_2}{1+\gamma_2} \geq \frac{\gamma_1}{1+\delta_1}$ and $\gamma_2 \geq \delta_1$, which is also true, by definition.

When, then conclude, that $\gamma_2 \leq \delta_1$ is a sufficient condition for $(1,3) \triangleright (3,3)$

□

From the previous proposition, we can deduce a necessary condition for $(3,3)$ outperforming both other regimes $(1,3)$ and $(3,1)$: $\gamma_1 \leq \delta_2$ and $\gamma_2 \leq \delta_1$. We leave to the reader to verify, that the previous also implies $\frac{\gamma_1}{1+\delta_1} \leq \frac{\delta_2}{1+\gamma_2}$ and $\frac{\gamma_2}{1+\delta_2} \leq \frac{\delta_1}{1+\gamma_1}$. Under such conditions, the performances of each regime, can be simplified as:

$$R(1,3) = \log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right)+\log_2(1+\gamma_2)$$

$$R(3,1) = \log_2\left(1+\frac{\gamma_2}{1+\delta_2}\right)+\log_2(1+\gamma_1)$$

$$\begin{aligned} R(3,3) \quad &= \log_2\left(1+\min\left[\gamma_1, \tfrac{\delta_2}{1+\gamma_2}\right]\right) \\ &+ \log_2\left(1+\min\left[\gamma_2, \tfrac{\delta_1}{1+\gamma_1}\right]\right) \end{aligned}$$

According to the expression of $R(3,3)$, 4 sub-cases have to be considered (2 for each min term). After a quick study, one can define an equivalent criterion for $(3,3) \triangleright (1,3)$ and $(3,3) \triangleright (3,1)$.

Proposition 8.2. *When $\gamma_2 \leq \delta_1$ and $\gamma_1 \leq \delta_2$, the two following statements hold:*

$$\left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2) \Leftrightarrow (3,3) \triangleright (1,3)$$

$$\left(1+\frac{\delta_1}{1+\gamma_1}\right)(1+\delta_2) \geq (1+\gamma_1)(1+\gamma_2) \Leftrightarrow (3,3) \triangleright (3,1)$$

Proof. In this proof, we focus only on the study of $(3,3) \triangleright (1,3)$. In a symmetric way, the second statement can be demonstrated.

Let us first focus on the cases where $\gamma_1 \leq \frac{\delta_2}{1+\gamma_2}$ or $\gamma_2 \leq \frac{\delta_1}{1+\gamma_1}$. For those configurations, the expression of $R(3,3)$ is rather simple and can be easily compared to those of (1,3) and (3,1), showing immediately that $(3,3) \triangleright (1,3)$. The main difficulty lies in the case where $\frac{\gamma_1}{1+\delta_1} \leq \frac{\delta_2}{1+\gamma_2} \leq \gamma_1 \leq \delta_2$ and $\frac{\gamma_2}{1+\delta_2} \leq \frac{\delta_1}{1+\gamma_1} \leq \gamma_2 \leq \delta_1$. In this last configuration, we get:

$$
\begin{aligned}
(3,3) \triangleright (1,3) &\Leftrightarrow R(3,3) - R(1,3) \geq 0 \\
\Leftrightarrow \frac{(1+\gamma_2+\delta_2)(1+\gamma_1+\delta_1}{(1+\gamma_1)(1+\gamma_2)} &\geq \frac{(1+\gamma_2)(1+\gamma_1+\delta_1)}{(1+\delta_1)} \\
\Leftrightarrow \left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) &\geq (1+\gamma_1)(1+\gamma_2)
\end{aligned}
$$

A necessary and sufficient condition for $(3,3) \triangleright (1,3)$ is then given by $\gamma_1 \leq \frac{\delta_2}{1+\gamma_2}$ or $\gamma_2 \leq \frac{\delta_1}{1+\gamma_1}$ or $\left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2)$. However, this sufficient condition can be simplified, since $\gamma_1 \leq \frac{\delta_2}{1+\gamma_2}$ or $\gamma_2 \leq \frac{\delta_1}{1+\gamma_1} \Rightarrow \left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2)$. Indeed,

$$
\begin{cases} \gamma_1 \leq \frac{\delta_2}{1+\gamma_2} \\ \gamma_2 \leq \delta_1 \end{cases} \Rightarrow \begin{cases} \frac{(1+\frac{\delta_2}{1+\gamma_2})}{1+\gamma_1} \geq 1 \\ \frac{1+\gamma_2}{1+\delta_1} \leq 1 \end{cases}
$$

$$
\Rightarrow \left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2)
$$

In a similar way

$$
\gamma_2 \leq \frac{\delta_1}{1+\gamma_1} \Rightarrow \frac{(1+\gamma_2)}{(1+\delta_1)} \leq \frac{(1+\gamma_1+\delta_1)}{(1+\gamma_1)(1+\delta_1)} = \frac{1+\frac{\gamma_1}{1+\delta_1}}{(1+\gamma_1)}
$$

And since, $\frac{\gamma_1}{1+\delta_1} \leq \frac{\delta_2}{1+\gamma_2}$, we get $\frac{(1+\gamma_2)}{(1+\delta_1)} \leq \frac{(1+\frac{\delta_2}{1+\gamma_2})}{(1+\gamma_1)}$. Which, means $\left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2)$.

In the end, we get that, when $\gamma_2 \leq \delta_1$ and $\gamma_1 \leq \delta_2$, $\left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2) \Leftrightarrow (3,3) \triangleright (1,3)$.

□

8.7 Proof of Proposition 5.4

In this section, we focus on defining the SNRs/INRs region in which (1,1) outperforms all the other interferences regimes.

Proposition 8.3.

$$(1,1)\ \textit{best regime}\ \Leftrightarrow \gamma_1 \geq \delta_2(1+\delta_1)\ \textit{and}\ \gamma_2 \geq \delta_1(1+\delta_2)$$

Proof. From Appendices 8.5 and 8.6, we know, that:

$$\begin{cases} (1,1) \rhd (1,3) \\ (1,1) \rhd (3,1) \\ (1,1) \rhd (3,3) \end{cases} \Leftrightarrow \begin{cases} \gamma_1 \geq \delta_2(1+\delta_1) \\ \gamma_2 \geq \delta_1(1+\delta_2) \\ (1,1) \rhd (3,3) \end{cases} \tag{8.19}$$

We also know that a sufficient condition for $(1,3) \rhd (3,3)$ is $\gamma_2 \geq \delta_1$. Respectively, a sufficient condition for $(3,1) \rhd (3,3)$ is $\gamma_1 \geq \delta_2$ From the previous, we notice that $(1,1) \rhd (1,3)$ does imply $(1,3) \rhd (3,3)$, and by transitivity, we can state that $(1,1) \rhd (3,3)$. From this, we can state that:

$$(1,1)\text{ best regime } \Leftrightarrow \begin{cases} (1,1) \rhd (1,3) \\ (1,1) \rhd (3,1) \end{cases} \Leftrightarrow \begin{cases} \gamma_1 \geq \delta_2(1+\delta_1) \\ \gamma_2 \geq \delta_1(1+\delta_2) \end{cases}$$

□

8.8 Proof of Proposition 5.5

In this section, we focus on defining the SNRs/INRs region in which (3,3) outperforms all the other interferences regimes.

Proposition 8.4. *A necessary and sufficient condition for (3,3) outperforming all the other regimes is:*

$$\begin{cases} \gamma_2 \leq \delta_1 \\ \textit{and}\ \gamma_1 \leq \delta_2 \\ \textit{and}\ \left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2) \\ \textit{and}\ \left(1+\frac{\delta_1}{1+\gamma_1}\right)(1+\delta_2) \geq (1+\gamma_1)(1+\gamma_2) \end{cases} \tag{8.20}$$

If any of those four conditions are not verified, then $(3,3)$ *is outperformed by either* $(1,3)$ *or* $(3,1)$.

Proof. From the previous Propositions 8.1 and 8.2, we know that,

- a sufficient condition for $(1,3) \rhd (3,3)$ is $\gamma_2 \geq \delta_1$.
- a sufficient condition for $(3,1) \rhd (3,3)$ is $\gamma_1 \geq \delta_2$.

- when $\gamma_2 \leq \delta_1$ and $\gamma_1 \leq \delta_2$, $\left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \geq (1+\gamma_1)(1+\gamma_2) \Leftrightarrow (3,3) \triangleright (1,3)$.

- when $\gamma_2 \leq \delta_1$ and $\gamma_1 \leq \delta_2$, $\left(1+\frac{\delta_1}{1+\gamma_1}\right)(1+\delta_2) \geq (1+\gamma_1)(1+\gamma_2) \Leftrightarrow (3,3) \triangleright (3,1)$.

At last, it is also easy to verify, using Equations (8.19), that $\gamma_2 \leq \delta_1$ and $\gamma_1 \leq \delta_2$ is sufficient for $(3,3) \triangleright (1,1)$. □

8.9 Proof of Proposition 5.6

From the previous Propositions 8.3 and 8.4, we can define the SNRs/INRs regions where $(1,1)$ and $(3,3)$ are the regimes granting the best achievable spectral efficiency. However, it remains a set of values $(\gamma_1, \gamma_2, \delta_1, \delta_2)$, that we denote Ω', for which both $(1,1)$ and $(3,3)$ are outperformed by either $(1,3)$ or $(3,1)$. In those configurations, we have to compare the performance of the two remaining regimes (1,3) and (3,1), in order to determine which one performs the best.

Proposition 8.5. *If* $(\gamma_1, \gamma_2, \delta_1, \delta_2) \in \Omega'$*, then*

- $\gamma_1 \leq \delta_2(1+\delta_1)$ *and* $\gamma_2 \geq \delta_1(1+\delta_2)$

- $\gamma_2 \geq \delta_1$ *and* $\gamma_1 \leq \delta_2$

are two sufficient conditions for $(1,3) \triangleright (3,1)$.

In a similar way, if $(\gamma_1, \gamma_2, \delta_1, \delta_2) \in \Omega'$*, then*

- $\gamma_1 \geq \delta_2(1+\delta_1)$ *and* $\gamma_2 \leq \delta_1(1+\delta_2)$

- $\gamma_2 \leq \delta_1$ *and* $\gamma_1 \geq \delta_2$

are two sufficient conditions for $(3,1) \triangleright (1,3)$.

Proof. From the previous results, one can obtain sufficient conditions by using the transitivity property of the $\triangleright$ operator. The proof is given for $(1,3) \triangleright (3,1)$, but can be easily transposed to $(3,1) \triangleright (1,3)$.

A sufficient condition for $(1,3) \triangleright (3,1)$ is $(1,3) \triangleright (1,1)$ and $(1,1) \triangleright (3,1)$, which is strictly equivalent to the first statement, according Equations (8.19). The same way, a sufficient condition for $(1,3) \triangleright (3,1)$ is $(1,3) \triangleright (3,3)$ and $(3,3) \triangleright (3,1)$, which is strictly equivalent to the second statement. □

With Proposition 8.5, we have covered all Ω' expect two regions, denoted Ω_A and Ω_B:

A. $\Omega_A = \Big\{\gamma_1,\gamma_2,\delta_1,\delta_2 \mid \frac{\gamma_1}{1+\delta_1} \le \frac{\delta_2}{1+\gamma_2} \le \gamma_1 \le \delta_2$ and $\frac{\gamma_2}{1+\delta_2} \le \frac{\delta_1}{1+\gamma_1} \le \gamma_2 \le \delta_1$ and $\left(1+\frac{\delta_2}{1+\gamma_2}\right)(1+\delta_1) \le (1+\gamma_1)(1+\gamma_2)$ and $\left(1+\frac{\delta_1}{1+\gamma_1}\right)(1+\delta_2) \le (1+\gamma_1)(1+\gamma_2)\Big\}$.

B. $\Omega_B = \Big\{\gamma_1,\gamma_2,\delta_1,\delta_2 \mid \frac{\delta_2}{1+\gamma_2} \le \frac{\gamma_1}{1+\delta_1} \le \gamma_1 \le \delta_2$ and $\frac{\delta_1}{1+\gamma_1} \le \frac{\gamma_2}{1+\delta_2} \le \gamma_2 \le \delta_1\Big\}$

For these two remaining regions A. and B., we formulate two propositions.

Proposition 8.6. *When* $(\gamma_1,\gamma_2,\delta_1,\delta_2) \in \Omega_B$,

$$(1,3) \triangleright (3,1) \Leftrightarrow \gamma_2 \ge \gamma_1 + (\delta_1 - \delta_2)$$

Proof. When $(\gamma_1,\gamma_2,\delta_1,\delta_2) \in \Omega_B$,

$$\begin{aligned} R(1,3) &= \log_2(1+\gamma_2) + \log_2\left(1+\frac{\delta_2}{1+\gamma_2}\right) = \log_2(1+\gamma_2+\delta_2) \\ R(3,1) &= \log_2(1+\gamma_1) + \log_2\left(1+\frac{\delta_1}{1+\gamma_1}\right) = \log_2(1+\gamma_1+\delta_1) \end{aligned}$$

From which, we immediately get:

$$(1,3) \triangleright (3,1) \Leftrightarrow R(1,3) \ge R(3,1) \Leftrightarrow \gamma_2+\delta_2 \ge \gamma_1+\delta_1$$

□

Proposition 8.7. *When* $(\gamma_1,\gamma_2,\delta_1,\delta_2) \in \Omega_A$,

$$(1,3) \triangleright (3,1) \Leftrightarrow (1+\gamma_1+\delta_1)\gamma_2\delta_2 \ge (1+\gamma_2+\delta_2)\gamma_1\delta_1$$

Proof. When $(\gamma_1,\gamma_2,\delta_1,\delta_2) \in \Omega_A$

$$\begin{aligned} R(1,3) &= \log_2(1+\gamma_2) + \log_2\left(1+\frac{\gamma_1}{1+\delta_1}\right) \\ R(3,1) &= \log_2(1+\gamma_1) + \log_2\left(1+\frac{\gamma_2}{1+\delta_2}\right) \end{aligned}$$

By definition,

$$\begin{aligned} &(1,3) \triangleright (3,1) \Leftrightarrow R(1,3) \ge R(3,1) \\ &\Leftrightarrow \frac{(1+\gamma_2)(1+\gamma_1+\delta_1)}{(1+\delta_1)} \ge \frac{(1+\gamma_1)(1+\gamma_2+\delta_2)}{(1+\delta_2)} \\ &\Leftrightarrow (1+\gamma_1+\delta_1)\gamma_2\delta_2 \ge (1+\gamma_2+\delta_2)\gamma_1\delta_1 \end{aligned}$$

□

By summin up the previous results, we get that when $(\gamma_1, \gamma_2, \delta_1, \delta_2) \in \Omega'$,

$$
(1,3) \text{ Best regime } \Leftrightarrow
\begin{cases}
[\gamma_2 \geq \delta_1 \text{ and } \gamma_2 \geq \gamma_1 + (\delta_1 - \delta_2)] \\
\text{ or } [\gamma_2 \leq \delta_1 \text{ and } (1+\gamma_1+\delta_1)\gamma_2\delta_2 \geq (1+\gamma_2+\delta_2)\gamma_1\delta_1]
\end{cases}
$$

8.10 Proof of Proposition 6.1

Let us now recall $(2,3)$ and $(3,2)$, two regimes defined previously/ Also, $(2,3)^*$ and $(3,2)^*$ are just symmetric versions of it. On one side, the interferer is able to decode the interference and cancel it out via SIC techniques. This first interferer transmits at its point-to-point channel capacity. On the other side, the interferer only transmits using half of the spectral resources, and is affected by interference. In Chapter 5, we have shown that these two regimes were outperformed by either $(3,1)$ or $(1,3)$. Let us now define the performance of $(3,2)$ and $(2,3)^*$ as:

$$
R((3,2),\omega) = \log_2\Big(1+\gamma(1,1)\Big)
$$

$$
+\frac{1}{2}\min\left[\log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right), \log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right)\right]
$$

$$
R((2,3),\omega)^* \log_2\Big(1+\gamma(1,2)\Big)
$$

$$
+\frac{1}{2}\min\left[\log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right), \log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right)\right]
$$

It is easy to verify that a sufficient condition for $(3,2) \triangleright (2,2)^1$ is $\gamma(1,1) \geq \gamma(1,2)$. The same way, a sufficient condition for $(2,3)^* \triangleright (2,2)^1$ is $\gamma(1,1) \leq \gamma(1,2)$. From this, we easily show that $(2,2)^1$ is always outperformed by either $(3,2)$ or $(2,3)^*$. Note that we have also shown, in Appendix 8.4, that for any channel configuration ω, the regimes $(2,3)$ and $(3,2)$ were outperformed by $(1,3)$ and $(3,1)$.

In a symmetric way, we can demonstrate that $(2,2)^2$ is outperformed by either $(3,2)^*$, $(2,3)$.

8.11 Proof of Proposition 6.2

Let us first define the spectral efficiencies associated to $(3,3)$ and $(1,1)^*$, as:

$$R((1,1)^*,\omega) = \log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right)+\log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right)$$

$$R((3,3),\omega) = \log_2\left(1+\min\left[\gamma(1,1),\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)$$
$$+\log_2\left(1+\min\left[\gamma(2,2),\frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right)$$

Since

$$\begin{cases} \min\left[\gamma(2,2),\frac{\gamma(2,1)}{1+\gamma(1,1)}\right] \leq \frac{\gamma(2,1)}{1+\gamma(1,1)} \\ \min\left[\gamma(1,1),\frac{\gamma(1,2)}{1+\gamma(2,2)}\right] \leq \frac{\gamma(1,2)}{1+\gamma(2,2)} \end{cases}$$

, it comes immediately that $R((1,1)^*,\omega) \geq R((3,3),\omega)$, i.e. $(1,1)^*\triangleright(3,3)$. In a symmetric way, the second part of the statement can be demonstrated.

8.12 Proof of Proposition 6.3

We first introduce 4 Propositions, that states conditions for which one regime might outperform another.

8.12.1 Conditions on Regimes Outperforming Other Regimes

Proposition 8.8. *The following statements hold*

- $(1,3)\triangleright(3,1)^* \Leftrightarrow \gamma(2,2) \geq \gamma(2,1)$
- $(3,1)\triangleright(1,3)^* \Leftrightarrow \gamma(1,1) \geq \gamma(1,2)$

Based on Table 6.1, it immediately comes, $(1,3)\triangleright(3,1)^* \Leftrightarrow R((1,3),\omega) \geq R((3,1)^*,\omega) \Leftrightarrow \log_2\left(1+\gamma(2,2)\right) \geq \log_2\left(1+\gamma(2,1)\right) \Leftrightarrow \gamma(2,2) \geq \gamma(2,1)$.
The same way, we can show that $(3,1)\triangleright(1,3)^* \Leftrightarrow \gamma(1,1) \geq \gamma(1,2)$.

Proposition 8.9. *The following statements hold*

- $(1,1)\triangleright(1,1)^* \Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(1,2))(1+\gamma(2,1))$.

- *A sufficient condition for* $(1,1)\triangleright(1,1)^*$ *is* $\gamma(1,1) \geq \gamma(1,2)$ *and* $\gamma(2,2) \geq \gamma(2,1)$.

- *A sufficient condition for* $(1,1)^*\triangleright(1,1)$ *is* $\gamma(1,1) \leq \gamma(1,2)$ *and* $\gamma(2,2) \leq \gamma(2,1)$.

Based on Table 6.1, it immediately comes,

$$\begin{aligned}
&(1,3)\triangleright(3,1)^* \\
&\Leftrightarrow R((1,1),\omega) \geq R((1,1)^*,\omega) \\
&\Leftrightarrow \log_2\left(\frac{(1+\gamma(1,1)+\gamma(2,1))(1+\gamma(2,2)+\gamma(1,2))}{(1+\gamma(2,1))(1+\gamma(1,2))}\right) \\
&-\log_2\left(\frac{(1+\gamma(1,1)+\gamma(2,1))(1+\gamma(2,2)+\gamma(1,2))}{(1+\gamma(1,1))(1+\gamma(2,2))}\right) \geq 0 \\
&\Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(1,2))(1+\gamma(2,1))
\end{aligned}$$

Then, it is easy to verify that $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$ (resp. $\gamma(1,1) \leq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$) are sufficient conditions for $(1,1)\triangleright(1,1)^*$ (resp. $(1,1)^*\triangleright(1,1)$).

Proposition 8.10. *The following statements hold*

- $(1,1)\triangleright(1,3) \Leftrightarrow \gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$.

- *A sufficient condition for* $(1,1)\triangleright(1,3)$ *is* $\gamma(1,1) \geq \gamma(1,2)$.

- $(1,1)\triangleright(3,1) \Leftrightarrow \gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$.

- *A sufficient condition for* $(1,1)\triangleright(3,1)$ *is* $\gamma(1,1) \geq \gamma(2,1)$.

- $(1,1)^*\triangleright(1,3)^* \Leftrightarrow \gamma(2,1) \geq \gamma(2,2)(1+\gamma(1,1))$.

- *A sufficient condition for* $(1,1)^*\triangleright(1,3)^*$ *is* $\gamma(2,1) \geq \gamma(2,2)$.

- $(1,1)^*\triangleright(3,1)^* \Leftrightarrow \gamma(1,2) \geq \gamma(1,1)(1+\gamma(2,2))$.

- *A sufficient condition for* $(1,1)^*\triangleright(3,1)^*$ *is* $\gamma(1,2) \geq \gamma(1,1)$.

Based on Table 6.1, we have:

$$\begin{aligned}&(1,1)\triangleright(1,3)\\&\Leftrightarrow \log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right)+\log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right)\\&-\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)},\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)+\log_2(1+\gamma(2,2))\geq 0\end{aligned}$$

Note that, if $\frac{\gamma(1,1)}{1+\gamma(2,1)}\leq\frac{\gamma(1,2)}{1+\gamma(2,2)}$, then we can not have $(1,1)\triangleright(1,3)$. Necessarily $\frac{\gamma(1,1)}{1+\gamma(2,1)}\geq\frac{\gamma(1,2)}{1+\gamma(2,2)}$ and $\log_2\left(1+\min\left[\frac{\gamma(1,1)}{1+\gamma(2,1)},\frac{\gamma(1,2)}{1+\gamma(2,2)}\right]\right)=\log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right)$. It follows:

$$\begin{aligned}&(1,1)\triangleright(1,3)\\&\Leftrightarrow \log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right)+\log_2\left(1+\frac{\gamma(2,2)}{1+\gamma(1,2)}\right)\\&-\log_2(1+\gamma(2,2)+\gamma(1,2))\geq 0\\&\Leftrightarrow \log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right)-\log_2(1+\gamma(1,2))\geq 0\\&\Leftrightarrow \frac{\gamma(1,1)}{1+\gamma(2,1)}\geq\gamma(1,2)\end{aligned}$$

Also, it is easy to verify that $\gamma(1,1)\geq\gamma(1,2)\Rightarrow\gamma(1,1)\geq\gamma(1,2)(1+\gamma(2,1))\Leftrightarrow(1,1)\triangleright(1,3)$.

The same way, using symmetric properties, the 6 other statements can be demonstrated.

Proposition 8.11. *A sufficient condition for* $(1,3)\triangleright(3,1)$ *is* $\gamma(2,2)\geq\gamma(2,1)(1+\gamma(1,2))$ *and* $\gamma(1,1)\geq\gamma(1,2)(1+\gamma(2,1))$.

Obtained by transitivity from Proposition 8.10.

8.12.2 Proof of the '6-Configurations Interference Classification, Proposition 6.3

We decompose the proof of this proposition, in 4 subsections, one for each part of the proposition.

8.12.2.1 Proof of 1) in Proposition 6.3

When $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$, we have, according to Propositions 8.8 and 8.9:

$$\begin{cases} (1,1)\triangleright(1,1)^* \\ (3,1)\triangleright(1,3)^* \\ (1,3)\triangleright(3,1)^* \end{cases}$$

And, only 3 configurations can pretend to be the Best Performing Configuration (BPC), when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$: $(1,1)$, $(1,3)$ and $(3,1)$.

At first, it is easy to verify, using Proposition 8.10, that $(1,1)$ is the best performing configuration (BPC) if and only if $\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$ and $\gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$. The first statement is set.

When $(1,1)$ is not the BPC, i.e. $\gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2))$ or $\gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1))$, we may identify 3 cases and confront the two remaining configurations : $(1,3)$ and $(3,1)$.

- When $\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2))$ and $\gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1))$, we have, according to Proposition 8.10, $(1,1)\triangleright(3,1)$ and $(1,3)\triangleright(1,1)$. This immediately leads to $(1,3)$ BPC.

- When $\gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2))$ and $\gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1))$, we have, according to Proposition 8.10, $(1,1)\triangleright(1,3)$ and $(3,1)\triangleright(1,1)$. This immediately leads to $(3,1)$ BPC.

One case remains: $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$ and $\gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2))$ and $\gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1))$. It is easy to verify that under these conditions, we have $\frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)}$ and $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$.

It immediately follows:

$$\begin{aligned} &(1,3)\triangleright(3,1) \\ &\Leftrightarrow \log_2(1+\gamma(2,2)+\gamma(1,2)) \geq \log_2(1+\gamma(1,1)+\gamma(2,1)) \\ &\gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1) \end{aligned}$$

Summing up, $(1,3)$ is BPC when

$$\left\{\begin{array}{l} \gamma(1,1) \geq \gamma(1,2) \\ \text{and } \gamma(2,2) \geq \gamma(2,1) \\ \text{and } \gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1)) \\ \text{and } [\gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2)) \\ \text{or } \Big(\gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2)) \\ \quad \text{and } \gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1)\Big)\Big] \end{array}\right.$$

Also, when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \geq \gamma(2,1)$, we have:

$$\left\{\begin{array}{l} \gamma(2,2) \geq \gamma(2,1)(1+\gamma(1,2)) \text{ and } \gamma(1,1) \leq \gamma(1,2)(1+\gamma(2,1)) \\ \Rightarrow \gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1) \end{array}\right.$$
$$\left\{\begin{array}{l} \gamma(2,2) \leq \gamma(2,1)(1+\gamma(1,2)) \text{ and } \gamma(1,1) \geq \gamma(1,2)(1+\gamma(2,1)) \\ \Rightarrow \gamma(2,2)+\gamma(1,2) \leq \gamma(1,1)+\gamma(2,1) \end{array}\right.$$

This leads to $(1,3)$ is BPC when

$$\left\{\begin{array}{l} \gamma(1,1) \geq \gamma(1,2) \\ \text{and } \gamma(2,2) \geq \gamma(2,1) \\ \text{and } (1,1) \text{ not BPC} \\ \text{and } \gamma(2,2)+\gamma(1,2) \geq \gamma(1,1)+\gamma(2,1) \end{array}\right.$$

This corresponds exactly to the second statement. The same way, we define the BPC conditions for $(3,1)$ and conclude the proof.

8.12.2.2 Proof of 2) in Proposition 6.3

We may proceed as in Subsection 8.12.2.1 and exploit symmetric properties.

8.12.2.3 Proof of 3) in Proposition 6.3

When $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$, we have, according to Propositions 8.8 and 8.9:

$$\begin{cases} \gamma(2,1)(1+\gamma(1,2)) \geq \gamma(2,2) \\ \gamma(1,1)(1+\gamma(2,2)) \geq \gamma(1,2) \\ (3,1)^{*} \triangleright (1,1)^{*} \\ (3,1) \triangleright (1,1) \\ (3,1) \triangleright (1,3)^{*} \\ (3,1)^{*} \triangleright (1,3) \end{cases}$$

At this point, only 2 configurations can pretend to be the Best Performing Configuration (BPC), when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$: $(3,1)^{*}$ and $(3,1)$. Also, note that when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$, we have necessarily $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ or $\frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}$.

We may now identify 3 cases and confront the two remaining configurations : $(1,3)$ and $(3,1)$.

1. If $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ and $\frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)}$, we have:

$$\begin{aligned} &(3,1) \triangleright (3,1)^{*} \\ &\Leftrightarrow R((3,1),\omega) \geq R((3,1)^{*},\omega) \\ &\Leftrightarrow log_2(1+\gamma(1,1)) + \log_2\left(1+\frac{\gamma(2,1)}{1+\gamma(1,1)}\right) \\ &-\log_2(1+\gamma(2,1)) - \log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right) \geq 0 \\ &\leftrightarrow \log_2\left(1+\frac{\gamma(1,1)}{1+\gamma(2,1)}\right) - -\log_2\left(1+\frac{\gamma(1,2)}{1+\gamma(2,2)}\right) \geq 0 \\ &\Leftrightarrow \frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \end{aligned}$$

Which is true, by definition. In this case, $(3,1)$ is BPC.

2. The same way, if $\frac{\gamma(1,1)}{1+\gamma(2,1)} \leq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ and $\frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}$, we have:

$$\begin{aligned} &(3,1) \triangleright (3,1)^{*} \\ &\Leftrightarrow \frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)} \end{aligned}$$

Which is not true, by definition. In this case, $(3,1)^{*}$ is BPC.

3. If $\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)}$ and $\frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}$, it follows:

$$\begin{aligned}
&(3,1) \triangleright (3,1)^* \\
&\Leftrightarrow \log_2\left(1+\frac{1+\gamma(1,1)}{1+\gamma(2,1)}\right) - \log_2\left(1+\frac{1+\gamma(1,2)}{1+\gamma(2,2)}\right) \geq 0 \\
&\Leftrightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2))
\end{aligned}$$

To sum up, $(3,1)$ is BPC if

$$\begin{aligned}
&\gamma(1,1) \geq \gamma(1,2) \\
&\text{and } \gamma(2,2) \geq \gamma(2,1) \\
&\text{and } \left[\left[\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ and } \frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)}\right]\right. \\
&\text{or } \left[\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ and } \frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}\right. \\
&\text{and } \left.\left. (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2))\right]\right]
\end{aligned}$$

That can be simplified into: $(3,1)$ is BPC if

$$\begin{aligned}
&\gamma(1,1) \geq \gamma(1,2) \\
&\text{and } \gamma(2,2) \geq \gamma(2,1) \\
&\text{and } \frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \\
&\text{and } (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2))
\end{aligned}$$

The same way: $(3,1)^*$ is BPC if

$$\begin{aligned}
&\gamma(1,1) \geq \gamma(1,2) \\
&\text{and } \gamma(2,2) \geq \gamma(2,1) \\
&\text{and } \frac{\gamma(2,1)}{1+\gamma(1,1)} \geq \frac{\gamma(2,2)}{1+\gamma(1,2)} \\
&\text{and } (1+\gamma(1,1))(1+\gamma(2,2)) \leq (1+\gamma(2,1))(1+\gamma(1,2))
\end{aligned}$$

Finally, when $\gamma(1,1) \geq \gamma(1,2)$ and $\gamma(2,2) \leq \gamma(2,1)$, we necessarily have:

$$\begin{cases}
\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ or } \frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)}
\end{cases}$$
$$\begin{cases}
\frac{\gamma(1,1)}{1+\gamma(2,1)} \geq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ and } \frac{\gamma(2,2)}{1+\gamma(1,2)} \geq \frac{\gamma(2,1)}{1+\gamma(1,1)} \\
\Rightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2))
\end{cases}$$
$$\begin{cases}
\frac{\gamma(1,1)}{1+\gamma(2,1)} \leq \frac{\gamma(1,2)}{1+\gamma(2,2)} \text{ and } \frac{\gamma(2,2)}{1+\gamma(1,2)} \leq \frac{\gamma(2,1)}{1+\gamma(1,1)} \\
\Rightarrow (1+\gamma(1,1))(1+\gamma(2,2)) \leq (1+\gamma(2,1))(1+\gamma(1,2))
\end{cases}$$

And this leads immediately to That can be simplified into: $(3, 1)$ is BPC if

$$\begin{aligned}&\gamma(1,1) \geq \gamma(1,2)\\ &\text{and } \gamma(2,2) \geq \gamma(2,1)\\ &\text{and } (1+\gamma(1,1))(1+\gamma(2,2)) \geq (1+\gamma(2,1))(1+\gamma(1,2))\end{aligned}$$

And $(3, 1)^*$ is BPC if

$$\begin{aligned}&\gamma(1,1) \geq \gamma(1,2)\\ &\text{and } \gamma(2,2) \geq \gamma(2,1)\\ &\text{and } (1+\gamma(1,1))(1+\gamma(2,2)) \leq (1+\gamma(2,1))(1+\gamma(1,2))\end{aligned}$$

8.12.2.4 Proof of 4) in Proposition 6.3

We may proceed as in Subsection 8.12.2.3 and exploit symmetric properties.

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